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COLLAPSE ANALYSIS FOR SHELLS OF GENERAL SHAPE: VOLUME II

USER'S MANUAL FOR THE STAGS-A COMPUTER CODE

LOCKHEED MISSILES AND SPACE COMPANY, INC.

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AIR FORCE FLIGHT DYNAMICS LABORATORY

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PALO ALTO, CALIFORNIA

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AIR FORCE FLIGHT DYNAMICS LABORATORY
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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13. ABSTRACT

This user's manual presents STAGS, a comprehensive computer code. STAGS is intended for the static analysis of arbitrary shells including the effects of nonlinearities caused by material behavior and finite deformations. Collapse loads based on nonlinear analysis can be computed as well as buckling loads based on classical bifurcation buckling theory with linear prestress. Arbitrary thermal and mechanical loadings can be specified. The manual provides instructions for use of the code and presents sample problems and solutions. The program is under development and the version presented here is expected to be updated in 1973.

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FOREWORD

This user's manual includes new features of the STAGS Computer Code developed by the Lockheed Palo Alto Research Laboratory, Palo Alto, California under sponsorship of the Lockheed Independent Research Program, NASA-Langley Research Center (NASA/LRC) (Contract No. NAS1-10843) and the Air Force Flight Dynamics Laboratory (AFFDL) Contract No. F33615-69-C-1523). The AFFDL Contract was administered under the Structures Division, with Mr. T. N. Bernstein (AFFDL/FBR) as Project Engineer.

This report was completed in October 1972 and covers work performed between April 1969 and October 1972. The supervision of this project was provided by Mr. B. O. Almroth of the Structural Mechanics Laboratory, LMSC.

This technical report has been reviewed and is approved.

Francis J. Janik, Jr.
Chief, Solid Mechanics Branch
Air Force Flight Dynamics Laboratory

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This user's manual presents STAGS, a comprehensive computer code. STAGS is intended for the static analysis of arbitrary shells including the effects of nonlinearities caused by material behavior and finite deformations. Collapse loads based on nonlinear analysis can be computed as well as buckling loads based on classical bifurcation buckling theory with linear prestress. Arbitrary thermal and mechanical loadings can be specified. The manual provides instructions for use of the code and presents sample problems and solutions. The theoretical basis for the program also is presented. The program is under development and the version presented here is expected to be updated in 1973.

CONTENTS

NOMENCLATURE

A, B, C	Matrices, see Eqs. (14), (19)
A, B, C	Coefficients of first fundamental form
C_{ij}	Stiffness coefficients, see Eq. (59)
D, E, F	Coefficients of second fundamental form
D	Stiffnesses, see Eq. (4)
E	Young's modulus
F	Vector of external forces
H	Parameter, see Eq. (58)
L	Operator
L'	Frechet derivative of L, see Eq. (11)
M	Moment resultant
N	Stress resultant
S	Stress vector
U	Strain energy
V	Total potential energy
W	Work done by external forces
X, Y, Z	Surface and normal coordinates used in Sections 5 and 6
X, Y	Vectors of displacement components
X*	Transpose of vector X
X_o	Vector of displacement components (linear solution)
Z	Strains or curvatures, see Eq. (4)
Z*	Transpose of vector Z
a	Parameter, see Eq. (45)
a^i	Area of subregion of shell surface
$a, b, \bar{a}, \bar{b}, c$	Parameters for elliptic cone, see Eq. (51)
$a_{i,j}, b_{i,j}$	Sides in rectangular region

$a^{\alpha\beta}$	Geometric tensor
$b_{\alpha\beta}$	Coefficients of second fundamental form
f	Function, see Eq. (31)
f,g,h	Functions, see Eq. (39)
g	Function, see Eq. (9)
h,k	Grid spacing in rectangular ref, see Eq. (36)
k	Ellipse ratio for yield surface
n	Normal-to-shell surface (vector)
t	Shell thickness
u,v,w	Displacement components
u_α	Covariant displacement component
x_n	Parameter, see Eq. (9)
x,y,z	Orthogonal Cartesian coordinates, see Eq. (39)
x_i	Coordinate of grid point, see Eq. (29)
$\bar{x}_i$	Coordinate of center of region, see Eq. (32)
$\alpha_{\alpha\beta}$	Thermal expansion coefficients
β	Rotation of edge
β_α, $\gamma_{\alpha\beta}$	Displacement gradients
γ	Shear strain
$\bar{\gamma}$	Shear strain at a reference surface
$\epsilon_{\alpha\beta}$	Strain tensor
ϵ	Inplane strain
$\bar{\epsilon}$	Inplane strain at a reference surface
$\Delta\epsilon_1, \Delta\epsilon_2, \Delta\gamma$	Strain increments

$\Delta\epsilon_1^P, \Delta\epsilon_2^P, \Delta\gamma^P$	Plastic strain increments
θ	Angle between coordinate lines
θ_i	Coordinate of grid point, see Eq. (29)
$\bar{\theta}_i$	Coordinate of center of region, see Eq. (32)
κ	Change of curvature
$\kappa_{\alpha\beta}$	Change of curvature tensor
λ	Multiplier, see Eq. (17)
ν	Poisson's ratio
ξ , η	Surface coordinates
σ , τ	Stresses
σ_1, σ_2, τ	Stresses at end of load step
$\bar{\sigma}_1, \bar{\sigma}_2, \bar{\tau}$	Stresses at beginning of load step
σ_T	Effective stress, see Eq. (25)
σ_Y	Yield strength for uniaxial tension
ϕ	Angular coordinate

INTRODUCTION

STAGS is a computer code developed to analyze the behavior of general shells under arbitrary static thermal and mechanical loading. Nonlinearities caused by material behavior and finite deformations are accounted for. The STAGS analysis is based on an energy formulation. Derivatives which appear in the energy expression are replaced by their two-dimensional finite difference approximations. When the energy is rendered stationary, the result is a system of nonlinear algebraic equations, which are solved by use of a modified Newton-Raphson method.

STAGS is an outgrowth of work on the buckling of cylindrical panels with nonuniform membrane stresses. This work was initiated at LMSC in 1963 under the sponsorship of NASA Marshall Space Flight Center (Ref. 1). The basic nonlinear computer program for cylindrical shells with cutouts (Ref. 2) and a linear version including analysis of free vibrations (Ref. 3) were developed under the LMSC Independent Research Program. Under contract with the Naval Ship Research and Development Center (NSRDC), the linear version of the code was developed to include shells of revolution with smooth but otherwise arbitrary cutouts (Ref. 4). This user's manual describes the latest version of STAGS which includes features developed during the last three years in separate studies performed under the sponsorship of the Air Force Flight Dynamics Laboratory (AFFDL), the Air Force Space and Missile Systems Organization (SAMSO), and the LMSC Independent Research Program. In the AFFDL effort, the nonlinear capability was extended to shells of more general shape and with cutouts of arbitrary contour. In addition, inelastic deformations were introduced and a capability to handle a finite difference grid with variable nodal point spacing was added. In a parallel effort sponsored under the LMSC Independent Research Program, the equations were generalized to include nonorthogonal coordinates (Ref. 5). Further expansion of STAGS was accomplished under the SAMSO sponsored study. Provisions were made to allow both

temperature and material properties to vary over the surface and through the thickness of the shell. A bifurcation buckling branch was added and parameter studies were made to evaluate the applicability of bifurcation buckling analysis for shells of general shape.

This user's manual describes features developed under each of these studies. For further information concerning the analyses and parameter studies, the reader is referred to final reports issued under these studies (Refs. 6 and 7).

This report begins with a discussion of collapse and bifurcation buckling. Then, a description of the scope and limitations of the code is given followed by details of the analysis. Next, the information necessary to use STAGS is presented.

Section 1
COLLAPSE OF SHELL STRUCTURES

A stress analyst usually defines the buckling load as the value of the load at which the fundamental branch of the load-displacement curve is intersected by a branch corresponding to a buckled equilibrium form. This definition is meaningful if the shell possesses a high degree of symmetry as in the case of axisymmetrically loaded shells of revolution. In reality, small deviations from the nominal geometry will destroy this symmetry. In this case, bifurcation in the equilibrium curve does not occur, and the classical concept of buckling exists only as an idealization.

It also may be noticed that the behavior exhibited by the shell as the buckling load is reached varies drastically from one case to another. As an example, buckling of a spherical cap under uniform pressure corresponds to a complete collapse of the shell. However, if the same spherical cap is subjected to a radial point force, buckling determined through a bifurcation analysis may correspond to a load level at which a gradual change in deformation pattern starts to develop. Loss of stability of the fundamental equilibrium configuration then corresponds only to an imperceptible change in the stiffness of the shell and is of no consequence to the designer.

In these cases, a nonlinear analysis of the axisymmetric prebuckling configuration is combined with a buckling analysis based on the assumption that the nonsymmetric displacements are infinitesimal. A two-dimensional nonlinear analysis gives a more complete description of the shell behavior but is much more expensive. Hence, for shells with a high degree of symmetry, the classical type of buckling analysis will remain an important instrument. Buckling in the classical sense means that the equilibrium of the fundamental branch becomes unstable relative to some deformation mode which is orthogonal to the prebuckling pattern. For a shell of a general shape, it can be expected that all deformation modes are present in the displacement pattern

corresponding to the fundamental branch. In this case, a complete nonlinear analysis is the only rigorous solution to the problem.

If a plane of symmetry exists with respect to loading as well as geometry, bifurcation is possible into modes which are antisymmetric relative to this plane. Thus, a nonlinear analysis which uses symmetry to reduce the amount of calculations should be supplemented by a bifurcation analysis with respect to antisymmetric displacements. Also, it is quite possible that a classical buckling analysis may give good estimates for the collapse load even in cases for which it is not strictly applicable. This would be the case if little change in load distribution or shell geometry occurs before a fatal type of buckling pattern starts to develop. For example, if a short cylinder is loaded by nonuniform external pressure, the bifurcation analysis may give a good approximation of the collapse load, but it may not represent an adequate solution for a longer cylinder.

Section 2
SCOPE, LIMITATIONS, PITFALLS

The computer program STAGS (Structural Analysis of General Shells) performs a nonlinear analysis of shells by use of a two-dimensional finite difference approach. Displacement and stress histories are computed corresponding to a given history of applied load, displacement, or temperature. Two branches of STAGS are described in this manual. The first branch is restricted to elastic material behavior and includes three sub-branches: (1) nonlinear collapse analysis, (2) linear analysis, and (3) buckling analysis based on the classical bifurcation approach with a linear prebuckling analysis. Collapse loads are found as limit points in the nonlinear load displacement curve. The program also is useful for postbuckling analysis of shells which behave according to classical buckling theory and for studies of the influence of imperfections.

The second branch is for inelastic material behavior but does not allow temperature or material properties to vary with any of the space coordinates. Only the nonlinear analysis sub-branch is included here.

STAGS applies to any shell for which a reference surface and a suitable set of gridlines can be mathematically defined. In general, the user of the program provides a subroutine describing the geometry, but several such routines for standard geometries are permanently included in the program (these are listed in Section 6). For the elastic branch the shell wall thickness can be varied, and elastic properties are allowed to vary with the shell coordinates and through the thickness. Cutouts in the shell wall and discrete eccentric stiffeners are included. The program is also general relative to boundary conditions and to loading. The loading can be applied in terms of variable surface tractions, point forces, or lineloads. Displacements, such as uniform end shortening of a cylindrical shell, can be applied if desired rather than fixed loading, and provision is made for thermal loading.

The general equations on which the analysis is based are given in Section 3.1 "Basic Equations," 3.3 "Bifurcation," and 3.4 "Plasticity." The finite difference expressions, valid for a variable grid spacing, are discussed in Section 3.5. The introduction of the finite difference approximations into the energy expression leads to a nonlinear algebraic equation system. Section 3.2 discusses the method of solution of this system.

Stiffeners and cutout edges must follow coordinate lines, or rather, the coordinate lines must be chosen so that they follow boundaries, internal or external, and the direction of stiffeners. This is not a severe program limitation because the capability of handling nonorthogonal grids has been included. However, analysis of more complicated shells will require some user skill.

Torsional stiffness and the resistance against rotation are included in the stiffener strain energy, but the energy caused by change of curvature in the plane of the shell has been omitted. Note that shell stiffeners usually are of the type shown in Fig. 2-1; i.e., they are designed to minimize bending of the shell surface. Because of the bending flexibility of the thin web, the line of intersection between the frame web and the shell surface can twist and bend (in the shell plane) without much resistance from the stiffener. In such a case, it is recommended that corresponding stiffnesses (J and I_z) are read in as zero. Then, in the stress output, the very small stresses caused by the changes of curvature in the plane of the shell will be ignored. For accurate analysis of stiffeners whose cross-section is likely to distort, it would be necessary to consider such stiffeners as shell branches.

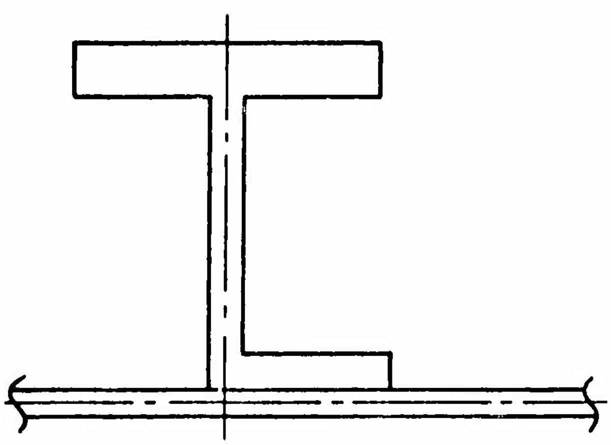


Fig. 2-1 Typical Stiffener

The input format requires that four shell edges be specified. For a shell that is closed in one direction, such as a complete shell of revolution, the program will assume that the shell is closed if appropriate boundary conditions are specified on opposite edges. For all

the special geometry routines presently in the program, the shells may be closed only in the η direction.

On each of the boundaries, the input parameters can be used to specify restraint on either of the three displacement components or on the rotation about the tangent to the edge. If displacement restraint is not specified for one or more of these quantities, the analysis will be based on appropriate natural boundary conditions (stress free). If more complicated boundary conditions are used, it will be necessary to modify the program in an area which is not easily accessible to the user. However, displacement restraints off the boundary lines can be introduced by using specified displacements in the load routine.

The most severe limitation of the program in practical analysis is dictated by computer economy. A large number of unknowns is generally unavoidable in a two dimensional numerical analysis regardless of the solution method used. Because of the coupling between the unknowns in two directions, the bandwidth of the resulting equation system is necessarily large in comparison to the bandwidth for one-dimensional problems. In addition, the nonlinearity of the formulation requires an iterative procedure in which the basic linear equation system is solved a number of times. Particularly if plasticity is included, the load step size must be small, and the convergence may be rather slow.

Two items are of special importance relative to the problem of computer runtime. One is the transformation of the structure into a model which is suitable for analysis; the other is the choice of strategy in the nonlinear analysis. Often, it is impossible to model the structure in a straightforward manner and considerable engineering skill may be needed. Eventually a few different models must be analyzed, all reflecting different types of local behavior. The strategy in the analysis involves choice of such items as stepsize and convergence criterion, and also involves the use of initial imperfections and the use of the results from the application of the bifurcation analysis. The choice of a proper strategy is very important for computer economy; this is discussed in detail in Section 4.

For maximum generality of the computer program it is necessary to introduce basic data (loads, temperature, thickness) through user-written subroutines. Such routines are discussed in Section 5. Usually, they are very easy to derive, but geometry routines may be complicated. Because of the importance of the strategy and of the modeling of the structure, it is concluded that considerable user skill is required to achieve the full potential of the STAGS program.

The basic problem dimensions (number of nodal points in each direction) are restricted only by the availability of mass storage. Some program limitations are imposed by dimension statements. In each direction, there can be as many as 80 stiffeners but only of 30 different types. The number of points for integration through the thickness must be an odd number and it may not exceed 9. In the plasticity analysis, as many as 10 material components may be used.

The unwary user may encounter pitfalls. For example, use of a too coarse grid may lead to inaccurate results. This problem is discussed in Section 4. Another problem arises when a buckling mode starts to develop, and its amplitude is small in comparison to the total displacement. The convergence criterion then may not be sufficiently sharp to catch the growth of this new deformation mode. This problem is discussed in Section 4. In particular, it is possible that the shell may buckle in a mode which is antisymmetric with respect to a plane about which symmetric behavior has been assumed. Such an occurrence probably will be revealed through an inspection of the buckling mode. In the bifurcation analysis, it is possible to use different boundary conditions for prebuckling and incremental displacements.

There is also a possibility that the problem as defined by the program user is not well posed. Difficulties will arise if the boundary conditions allow rigid body displacements. This will make the system extremely ill conditioned; it would be singular except for the truncation errors in the finite difference expressions. The difficulty will occur only in analysis of a free body which is subjected to a self-equilibrating force system. Rigid body displacements can be restrained by specifying displacement constraints as discussed in Section 6 (input description, L-1 and L-2 cards). Because of the aforementioned

truncation error, a small force may develop at restrained points. Therefore, it is advisable to fix points at which the structure is relatively stiff. Results would be meaningless if a load is defined on an edge at which the displacements are restrained in the direction of the load. The same would be the case if a prescribed displacement is in conflict with displacement boundary conditions on an adjacent edge.

For a general shell shape, the bifurcation buckling branch of the program does not correspond to a rigorous application of stability theory. The results may or may not represent good approximations. This problem is discussed in Ref. 3.

Section 3
ANALYSIS

3.1 BASIC EQUATIONS

For shells of a more general shape than the shell of revolution, it is not possible to separate the governing differential equations. The analysis thus requires the use of two independent space variables, and the numerical analysis is drastically encumbered. In addition, if the collapse load of the shell is to be determined, a nonlinear analysis is encountered. The STAGS program is based on a discretization of the total potential energy by use of finite difference expressions.

The energy method was used because it simplifies the handling of shell cutouts and discrete stiffening. For a general shell, the surface coordinates are chosen for practical reasons to coincide with shell boundaries. In this case, they are not necessarily lines of curvature, and the basic equations must be written in terms of non-orthogonal coordinates. For brevity, the basic equations are given here in tensorial form, but a formulation in terms of physical components is available in Ref. 2.

The expression for the strain energy is

$$U = \frac{1}{2}\frac{E}{1-\nu^2}\left[(1-\nu)\, a^{\alpha\beta} a^{\beta\lambda} + \nu\, a^{\alpha\rho} a^{\beta\lambda}\right]\left[t\, \epsilon_{\alpha\rho}\epsilon_{\beta\lambda} + \frac{t^3}{12}\kappa_{\alpha\rho}\kappa_{\beta\lambda}\right] \tag{1}$$

The expressions for strains and curvature changes are

$$\begin{aligned} \epsilon_{\alpha\beta} &= \frac{1}{2}(\gamma_{\alpha\beta} + \gamma_{\beta\alpha}) + \frac{1}{2}\beta_\alpha\beta_\beta + \frac{1}{2}\gamma_{\rho\alpha}\gamma^{\rho}_{.\beta} \\ \kappa_{\alpha\beta} &= \beta_\beta|_\alpha + b^\rho_\alpha\gamma_{\rho\beta} - b_{\alpha\beta}\gamma^{\rho}_{.\rho} \end{aligned} \tag{2}$$

where $\gamma_{\alpha\beta}$ and β_α are the displacement gradients defined by

$$\gamma_{\alpha\beta} = u_\alpha|_\beta - b_{\alpha\beta} w$$

$$\beta_\alpha = w,_\alpha + b_\alpha^\beta u_\beta \tag{3}$$

and the curvature tensor $b_{\alpha\beta}$ and the normal displacement w are defined with respect to the inner shell normal vector. The curvature tensor differs from that by Sanders (Ref. 8) because it is valid for larger out-of-plane rotations. A complete derivation is given in Ref. 9.

3.2 SOLUTION METHOD

The numerical solution is based upon a two-dimensional finite difference approximation. The shell surface is covered with mesh lines parallel to the coordinate lines, and the freedoms of the system are the normal displacements, w, at the grid points and the tangential displacements, u and v, at points between adjacent gridpoints.

After replacement of the displacement functions and their derivatives in the governing equations by finite difference approximations (see Section 3.5), the strain energy density at mesh station i can be written in the form

$$\Delta U^i = \frac{1}{2} Z^{i*} D^i Z^i \tag{4}$$

where D^i is a 6 × 6 positive definite matrix of constants and Z^i is a column vector of strain and curvature changes at station i. D^i and Z^i are functions of the geometric parameters of the shell; in addition, D^i is dependent on the material properties. Z^i is a nonlinear (quadratic) function of the displacement unknowns and thus ΔU^i is a fourth-order polynomial. The vector of stress resultants S^i at station i is given by

$$S^i = D^i Z^i \tag{5}$$

The total potential energy, V, of the shell is obtained by combination of the strain energy and the work done by the external forces,

$$V = U - W \tag{6}$$

where

$$U = \sum_{i}^{m} \Delta U^{i} \cdot a^{i}$$

and

$$W = X^{*} \cdot F$$

Here X denotes the vector of displacement components, F is the vector of external forces and a_i is the area of the i^{th} subregion. A necessary condition for static equilibrium is that the total potential energy be stationary. This condition requires the vanishing of the first variation of V and leads to the equation

$$LX = F \tag{7}$$

where the operator L is defined by

$$LX = \text{Grad } U \tag{8}$$

L is thus a "stiffness" operator which relates displacement components and external forces and is nonlinear in the general case.

When only linear terms are included in the definition of the strains and changes in curvature, L is a linear operator which may be readily represented in matrix form [see Section 3.3 and Eqs. (22) and (23)]. In this case, the matrix is positive definite (with the choice of proper boundary conditions) and Eq. (7)

$$LX = F$$

may be solved by one of many direct or iterative methods. However, when geometric nonlinearities (i.e., rotations) are included, L becomes a polynomial operator of third degree and iterative methods must be employed for solution of the equations. For a general collapse analysis, it is necessary to solve the operator equation, Eq. (7) for a sequence of applied loads. In fact, the only practical method consists of a sequence of load steps chosen so that the initial solution is nearly linear and subsequent solutions change only moderately from one step to the next. Such a procedure is mandatory for two reasons: first, the feasibility of the iterative methods of solution depend on reasonably good initial approximations and second, a reliable detection of collapse requires such a stepwise procedure because of the non-uniqueness of solutions to nonlinear equation systems.

Thus, at the i^{th} load step in a collapse analysis, the operator equation

$$LX = F_i \tag{8}$$

must be solved where F_i is the vector of constants generated by the applied load (mechanical, thermal, plastic pseudo loads).

A brief description of Newton's method and the modified Newton method for the case of a function of one variable displays the principal features of the methods used for the solution of the nonlinear equation in STAGS.

Newton's method for the solution of the problem $g(x) = 0$ (see Fig. 3-1) is defined by

$$x_{n+1} = x_n - g(x_n)/g'(x_n) \tag{9}$$

The iteration converges quadratically to the solution provided that the initial estimate x_0 is sufficiently accurate. A geometric interpretation of Newton's method is well known; from any point on the curve $g(x)$, the next approximation is obtained by extending the tangent to the curve at the point to the x-axis. In Fig. 3-2, the "modified" Newton iteration is illustrated. The method is defined by the equation

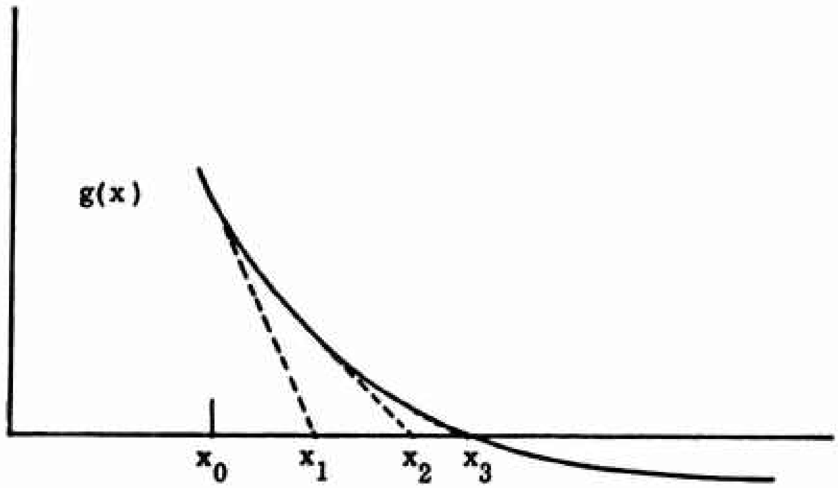


Fig. 3-1 Newton's Method

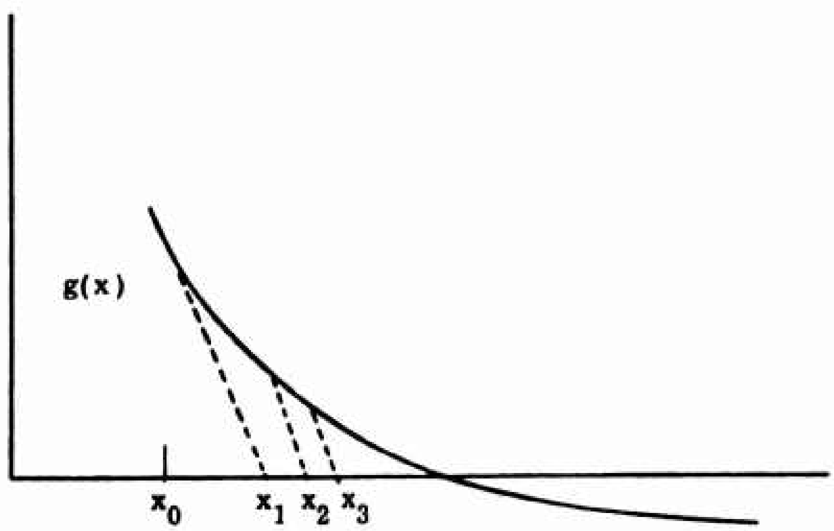


Fig. 3-2 Modified Newton Method

$$x_{n+1} = x_n - g(x_n)/g'(x_0) \tag{10}$$

Geometrically, this iteration corresponds to extending lines from the curve $g(x)$ to the axis which are all parallel to the tangent at x_0. While the convergence of the modified Newton method obviously is slower than that of the standard Newton method, it avoids repeated computation of $g'(x)$. The most effective strategy usually calls for periodic recomputation of $g'(x)$.

Both Newton's method and the modified Newton method have been generalized to n-dimensional Euclidean space.

The treatment of the complete nonlinear solution of Eq. (7) as well as of bifurcation buckling is facilitated by introduction of the concept of the derivative L' of L (Ref. 6). In particular, for the operator L, the derivative L' (sometimes called the Frechet derivative of L) is an n by n matrix whose elements are

$$L'_{i,j} = \frac{\partial^2 U}{\partial X_{(i)} \partial X_{(j)}} \tag{11}$$

Most of the elementary properties of ordinary derivatives also hold for the Frechet derivatives L' of an operator L.

Because L' is a function of a particular displacement vector X (unless the nonlinear terms are dropped) the Frechet derivative will usually be denoted L'_X to indicate this dependence. With the use of the derivative L' of the operator L, Newton's method may be readily generalized to obtain a solution of Eq. (7). The iteration is defined by

$$L'_{X_k} (X_{k+1} - X_k) = F - LX_k \tag{12}$$

If X_0 is sufficiently close to a solution X and if L'_X is not a singular matrix, the iteration converges to X. Under these assumptions, it also can be shown that the converged solution is unique in some neighborhood of X (Ref. 10).

Similarly, with the aid of the derivative L'_X, the modified Newton method may be applied to the operator equation Eq. (8). The general form of the iteration then becomes

$$L'_{X_m} (X_{n+1} - X_n) = F_i - LX_n \tag{13}$$

This iteration formula contains, as special cases, the most commonly used iterative methods. For example, if $X_m \equiv 0$, L'_{X_m} is identical with the matrix A corresponding to the linearization of the operator L. If B denotes that part of the operator L obtained by variation of the 3rd and 4th degree terms of V, the iteration may be written

$$AX_{n+1} = F_i - BX_n \tag{14}$$

This iteration corresponds to repeated solution of the linear equation system which is obtained by substitution of the previous iterates for the unknowns in the nonlinear terms. The method is often effective in cases for which the effects of nonlinear terms are small. Unfortunately in practical cases (e.g., a cylinder with a cutout), the iteration often fails to converge well before the collapse load is reached. Furthermore, when the method is applicable, a bifurcation analysis would frequently be an even more effective choice.

If only one iteration of Eq. (13) is performed for each load step and if $m = i$, the resulting method is equivalent to the frequently used "incremental analysis." Both this method, and the standard Newton method, require the recomputation of the derivative matrix and its factorization one or more times for each load step. For the "incremental analysis," the load steps must be chosen small enough to ensure accuracy, but the iteration does not itself provide the information necessary for such a determination.

In contrast to the iterative methods described above, the modified Newton method provides accurate solutions independent of the size of the load step (numerical error

does not accumulate) and at the same time avoids the necessity of frequent recomputation and factorization of the derivative matrix L'. This latter feature exploits a fundamental advantage of the finite difference treatment used here which permits extremely rapid evaluation of LX_n. Even when the best available methods of factorization are employed, the time required for a single evaluation of LX_n is many times less than the factorization time. The modified Newton method is employed in STAGS to combine rigorous results with the most economical computational effort. The effective use of the modified Newton method requires choices both as to the size of load steps and as to when the derivative L' should be recomputed and refactored. The STAGS program contains as much built-in decision making capability regarding these questions as appears feasible. However, it is still necessary for the user of the program to consider the best overall "strategy" relating to these choices (see Section 4).

3.3 BIFURCATION

Note that the mathematical characterization of bifurcation buckling also is provided by the generalized Newton method. Let X_O be a solution of Eq. (7) under a given vector F of external forces. If every neighborhood of X_O contains another vector Y which satisfies the equation

$$LY = F \tag{15}$$

then bifurcation is said to take place for the shell under the load F. From the previous remarks on the conditions for convergence of Newton's method to a unique solution, it follows that a necessary condition for bifurcation is that L'_{X_O} be a singular matrix,

$$\det(L'_{X_O}) = 0 \tag{16}$$

Classical bifurcation buckling theory may be obtained easily from Eq. (16). It is assumed that X_O may be written

$$X_0 = \lambda X_L \tag{17}$$

where X_L is the linear solution for a load vector F_L. Thus, Eq. (16) becomes

$$\det (L'_{\lambda X_L}) = 0 \tag{18}$$

Equation (18) is an algebraic eigenvalue problem of the form

$$\det (A - \lambda B - \lambda^2 C) = 0 \tag{19}$$

In classical bifurcation theory, the C matrix, which arises from the prebuckling rotations, is often omitted and the eigenvalue problem

$$AX = \lambda BX \tag{20}$$

is obtained.

When bifurcation exists but the prebuckling displacements are not linear, the solution of Eq. (16) generally requires a stepwise procedure. One such method is given by the equations

$$\det (\dot{L}'_{\lambda_{k+1}} X_k) = 0 \tag{21}$$

$$X_{k+1} = \lambda_{k+1} X_k$$

where X_0 is the linear solution. A sequence of eigenvalue problems is solved and, if the method is successful, λ_k approaches one. A nonlinear bifurcation treatment, equivalent to Eq. (21) was presented in Ref. 11 and has been used successfully to study a large variety of problems. For the two-dimensional problems under consideration

here, it appears that such methods may be nearly as costly as the complete nonlinear analysis available in STAGS. Consequently, only a classical bifurcation buckling analysis is implemented in the STAGS program.

The formation of the A and B matrices of Eq. (19) will be considered briefly. The elements of the Frechet derivative matrix $L_{\lambda}^{\prime}X_L$ (which define the matrices A and B) are determined according to Eq. (11). The rules for computing derivatives of polynomials are easily programmed, and the formation of the A and B matrices therefore is well suited to automatic treatment on the computer. Thus, for example, if $X_{(i)}$ and $X_{(j)}$ are the i^{th} and j^{th} displacement components, by use of Eq. (4), (5) and (6) the following is obtained:

$$\frac{\partial^2 U}{\partial X_{(i)} \partial X_{(j)}} = \sum_{k=1}^{m} a^k \frac{\partial^2 \Delta U^k}{\partial X_{(i)} \partial X_{(j)}} \tag{22}$$

The k^{th} term of this sum is

$$\frac{\partial^2 \Delta U^k}{\partial X_{(i)} \partial X_{(j)}} = \frac{\partial^2 Z^{k*}}{\partial X_{(i)} \partial X_{(j)}} \lambda S_L^k + \frac{\partial Z^{k*}}{\partial X_{(i)}} D^k \frac{\partial Z^k}{\partial X_{(j)}} \tag{23}$$

In the first term on the right hand side of Eq. (23), note that S_L^k is the linear stress resultant vector at station k and that only the quadratic terms (rotations) need be considered in forming the partial derivatives

$$\frac{\partial^2 Z^{k*}}{\partial X_{(i)} \partial X_{(j)}} .$$

Contributions from this term go into the B matrix. Assuming the prebuckling rotations may be neglected for the classical theory, the last term of Eq. (23) generates additions only to the A matrix. The A matrix then is identical with the linear stiffness matrix.

If the prebuckling rotations are included to treat nonlinear bifurcation, the last term of Eq. (23) generates a C matrix and provides additional contributions to the B matrix. In this case, the prebuckling stress resultant vector S would also include nonlinear terms.

In conclusion, it should be noted that the use of operation notation can be (and often is) avoided. For example, a nonlinear equation system can be obtained by writing the displacements at step $n + 1$ as the sum of the known displacements at step n plus an increment of displacement. Newton's method can be derived for functions of many variables by considering Taylor series expansions (see Ref. 12). Bifurcation buckling theory may be based on the theory of adjacent equilibrium states. In all of these cases, the development is rather complicated and burdened with excessive detail. The use of operator methods, however, permits the immediate application of a well developed theory. The concise operator notation facilitates manipulation and programming for the computer. In particular, the definition and computation of the basic matrices of Eq. (19) are greatly simplified. The recipe is outlined in Eq. (22) and (23) and can be performed using straightforward algebraic procedures. Finally, the relations between a complete nonlinear analysis, linear (classical) and nonlinear bifurcation theory and Newton's method are clarified.

3.4 PLASTICITY

Introduction of inelastic behavior has been done within the framework of the energy principle on which the elastic analysis was based. Essentially, the plastic deformations are considered as load terms; they are completely analogous to thermal expansions except that they are not known in advance. A series of elastic problems are solved by use of the energy principles in which the "load terms" are gradually modified until they correspond to the computed state of stress and to specified nonlinear stress strain relations at all points over the shell surface and through the shell thickness.

The plasticity theory used has been proposed by Besseling (Ref. 13), and is based on a principle which originally was suggested by White. This theory is very promising

because it is rather simple in its application yet retains such features as strain hardening and the Bauschinger effect.

The White-Besseling theory, as applied here, assumes that the material consists of several components which have identical elastic properties and exhibit ideal plasticity (no strain hardening) but have different yield strength. As the strain is the same in all components, the stress-strain curve will experience a decrease in slope as the stress reaches the yield limit for any one of the components; the respective components then cease to take additional load. The composite thus exhibits strain hardening with a piecewise linear stress-strain relation. Use of only one component will, of course, result in application of ideal plasticity theory. If the stress is reversed after loading beyond the yield limit for one or more components, yield will occur in the reversed direction at an average stress in the composite which is lower than the stress for original yield. This is demonstrated in the uniaxial stress-strain curve shown in Fig. 3-3. Tension is first applied, OAB, beyond the yield limit and followed by strain reversal, BCD, into the zone of yield in compression. The yield ellipse for the weakest component and the loading history in this component are also shown in this figure. Clearly, yield in compression will occur when the total strain is $(\epsilon_1 - 2\epsilon_y)$, i.e., the yield in compression occurs at a much lower stress if the material previously has been subjected to tension stresses above the yield point. To introduce the Bauschinger effect this way is appealing because it reflects the microstress theory which now generally is accepted as the explanation of the Bauschinger effect.

The White-Besseling plasticity theory is implemented in the computer program in the following manner:

(1) The inelastic behavior of the material is defined through specification of

- The number of components
- The relative volume of each component
- The yield strength for each component

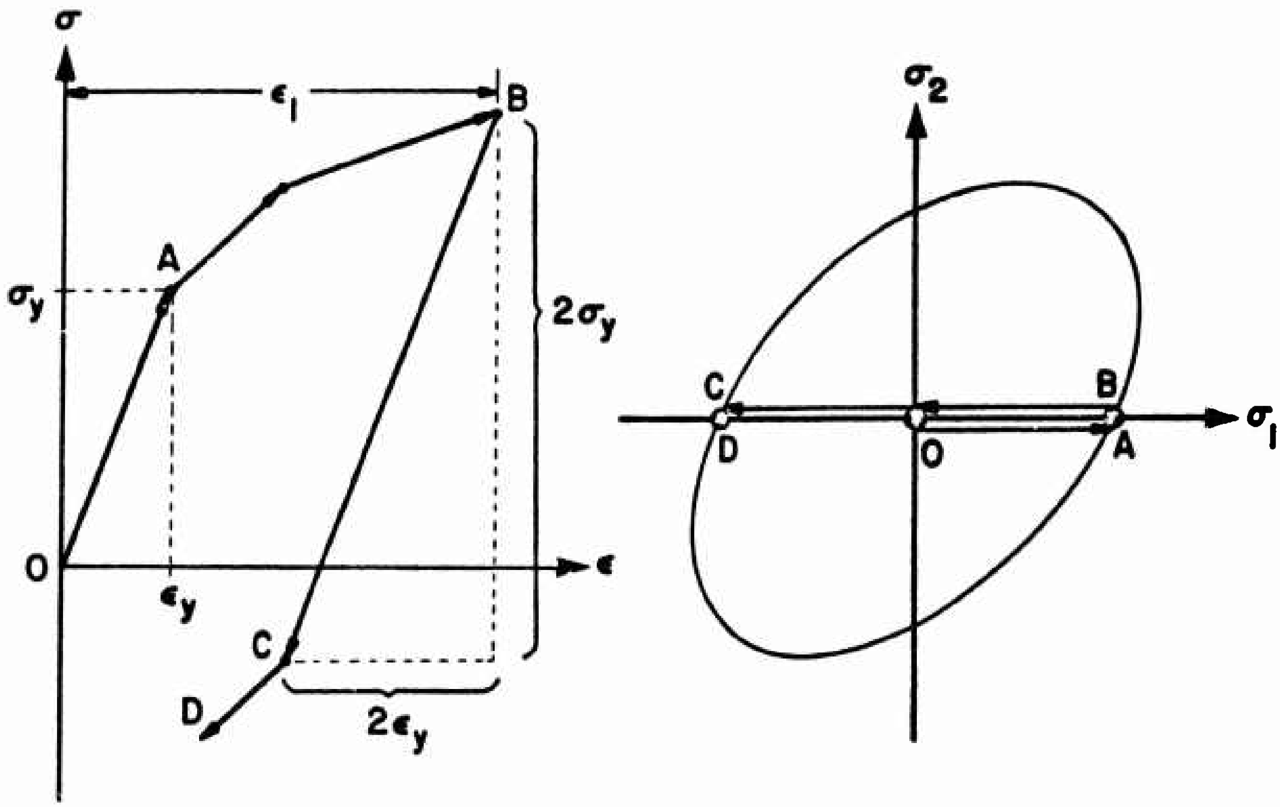


Fig. 3-3 Stress Strain Curve Showing Bauschinger Effect

(2) The strains are estimated for all points in the shell over the shell coordinates and through the thickness. This generally is done through extrapolation from previous solutions.

(3) A subroutine is called within which, for each of the material components, the stress corresponding to the assumed strains is determined. The total stress for the composite then is found.

(4) Once total strains and stresses are known, the plastic part of the strain increment can be determined and added as a pseudo-load in an elastic analysis.

(5) New strains are computed and used as estimates. The procedure is continued until the computed strains agree to within a given margin with the estimated strains.

The following operations are performed in the above-referenced subroutine:

(1) Information about material properties is transferred into the routine together with the estimated strain increments ($\Delta\epsilon_1$, $\Delta\epsilon_2$, and $\Delta\gamma$) and stresses at the end of the previous load step ($\bar{\sigma}_1$, $\bar{\sigma}_2$, $\bar{\tau}$).

(2) New stresses are computed under the assumption that the load step is elastic.

$$
\begin{aligned}
\sigma_1 &= \bar{\sigma}_1 + \frac{E}{1 - \nu^2} (\Delta\epsilon_1 + \nu\Delta\epsilon_2) \\
\sigma_2 &= \bar{\sigma}_2 + \frac{E}{1 - \nu^2} (\Delta\epsilon_2 + \nu\Delta\epsilon_1) \\
\tau &= \bar{\tau} + \Delta\gamma E/[2 (1 + \nu)]
\end{aligned}
\tag{24}
$$

(3) Set $\sigma_T^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 + k^2\tau^2$ (25)

where k is the ellipse ratio for the assumed yield surface (usually $\sqrt{3}$).

(4) If σ_T^2 is less than σ_Y^2, the load step is elastic in this component (loading or unloading). If this is the case for all components, the calculations for the load step are concluded. There are no pseudo loads caused by plastic strain increments.

(5) If σ_T^2 is larger than σ_Y^2 for some component, the step is at least partly inelastic for this component. As we have assumed ideal plasticity, the stresses can be computed from the conditions that

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 + k^2\tau^2 = \sigma_Y^2 \tag{26}$$

where

$$\sigma_1 = \bar{\sigma}_1 + \frac{E}{1 - \nu^2} \left[\Delta\epsilon_1 - \Delta\epsilon_1^p + \nu (\Delta\epsilon_2 - \Delta\epsilon_2^p)\right]$$

$$\sigma_2 = \bar{\sigma}_2 + \frac{E}{1 - \nu^2} \left[\Delta\epsilon_2 - \Delta\epsilon_2^p + \nu (\Delta\epsilon_1 - \Delta\epsilon_1^p)\right] \tag{27}$$

$$\tau = \bar{\tau} + \frac{E}{2(1 + \nu)} \left[\Delta\gamma - \Delta\gamma^p\right]$$

and that the plastic flow is perpendicular to the yield surface

$$\frac{\Delta\epsilon_1^p}{\Delta\epsilon_2^p} = \frac{2\bar{\sigma}_1 - \bar{\sigma}_2}{2\bar{\sigma}_2 - \bar{\sigma}_1} \quad ; \quad \frac{\Delta\epsilon_1^p}{\Delta\gamma^p} = \frac{2\bar{\sigma}_1 - \bar{\sigma}_2}{2k^2 \bar{\tau}} \tag{28}$$

After the stresses have been determined in the components, the average stress in the composite is found readily. As the elastic constants are the same for all components, the plastic part of the strain increment (i.e., the pseudo loads), can easily be obtained.

3.5 FINITE DIFFERENCE APPROXIMATIONS

The STAGS program includes a capability of using grids with variable spacing. The discretization of functions and their derivatives in such nets has been handled in the following manner.

Suppose a shell panel has been covered with a system of mesh lines whose coordinates are given by

$$x_i \quad , \quad i = 1, m$$

and (29)

$$\theta_j \quad , \quad j = 1, n$$

where x and θ are the axial and circumferential coordinates, respectively. Corresponding to each pair of values (i, j) a rectangular region $R_{i,j}$ is defined with sides of length

$$a_{i,j} \equiv 1/2 \, |x_{i+1} - x_{i-1}| \quad ,$$

$$b_{i,j} \equiv 1/2 \, |\theta_{j+1} - \theta_{j-1}| \quad . \tag{30}$$

The regions $R_{i,j}$ (and lengths $a_{i,j}$, $b_{i,j}$) are modified at boundaries of a shell by considering only those portions which would be within the panel. A double integral of a function f over the region R of the panel may then be approximated by

$$\iint_R f \, dx d\theta = \sum_{i=1}^{m} \sum_{j=1}^{n} f_{i,j} \, a_{i,j} \, b_{i,j} \tag{31}$$

The discretization is completed when the integrand functions $f_{i,j}$ are evaluated at the centroids of the regions $R_{i,j}$ in terms of the neighboring displacement components.

First, note that the tangential displacements u and v have been located at corners of the regions $R_{i,j}$. Furthermore, the energy expressions for a general shell include derivatives of u and v only up to the first order. Hence, even with arbitrary rectangular spacing, only central difference formulas for the u and v displacements are required. In contrast, the normal displacement w has been located at the mesh node points (x_i , θ_j), and more general finite difference formulas must be developed.

The coordinates of the centroid of a region $R_{i,j}$ are given by

$$\bar{x}_i = 1/4 \, (x_{i-1} + 2x_i + x_{i+1})$$

$$\bar{\theta}_j = 1/4 \, (\theta_{j-1} + 2\theta_j + \theta_{j+1}) \tag{32}$$

Variable spacing is considered first with respect to a single coordinate x only. With the help of a Taylor's expansion (or equivalently by the use of interpolation formulas), the difference formulas for w, $w,_x$ and $w,_{xx}$ at $\bar{x}_i$ may be established as

$$
\begin{aligned}
(w)_i \equiv w|_{\bar{x}_i} = {} & w_{i-1}/16 \cdot [(h - k) \cdot (3k + h)/(h^2 + hk)] \\
& + w_i/16 \cdot [(h + 3k) \cdot (3h + k)/h + k)] \\
& + w_{i+1}/16 \cdot [(k - h) \cdot (3h + k)/(hk + k^2)]
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
(w,_x)_i \equiv w,_x|_{\bar{x}_i} = {} & - w_{i-1}/(2h) \\
& + w_i [1/(2h) - 1/(2k)] \\
& + w_{i+1}/(2k)
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
(w,_{xx})_i \equiv w,_{xx}|_{\bar{x}_i} = {} & w_{i-1} \cdot 2/[h \cdot (h + k)] \\
& - w_i \cdot 2/(h \cdot k) \\
& + w_{i+1} \cdot 2/[k \cdot (h + k)]
\end{aligned}
\tag{35}
$$

where

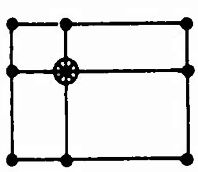
$$
\begin{aligned}
h &= x_i - x_{i-1} \\
k &= x_{i+1} - x_i
\end{aligned}
\tag{36}
$$

The corresponding formulas for the θ coordinate are obtained by appropriate substitutions and are denoted with superscripts

$$
\begin{aligned}
(w)^j &\equiv w|_{\bar{\theta}_j} \\
(w,_\theta)^j &\equiv w,_\theta|_{\bar{\theta}_j} \\
(w,_{\theta\theta})^j &\equiv w,_{\theta\theta}|_{\bar{\theta}_j}
\end{aligned}
\tag{37}
$$

The required two-dimensional difference formulas now are obtained by combining the formulas for both coordinate directions

$$\begin{aligned}
\bar{w}_{i,j} &\equiv w\big|_{(\bar{x}_i,\, \bar{\theta}_j)} = ((w)_i)^j = ((w)^j)_i \\
\overline{w,xx}_{i,j} &\equiv w,_{xx}\big|_{(\bar{x}_i,\, \bar{\theta}_j)} = ((w,_{xx})_i)^j \qquad (38) \\
\overline{w,\theta\theta}_{i,j} &\equiv w,_{\theta\theta}\big|_{(\bar{x}_i,\, \bar{\theta}_i)} = ((w,_{\theta\theta})^j)_i \\
\overline{w,x\theta}_{i,j} &\equiv w,_{x\theta}\big|_{(\bar{x}_i,\, \bar{\theta}_i)} = ((w,_x)_{i,\theta})^j
\end{aligned}$$



In general, these equations involve the 9 point "star" of neighboring values. However, it is easily seen that all of the formulas reduce to the standard central difference formulas when uniform rectangular spacing is used. All of the difference formulas are exact when the displacement function w behaves quadratically.

3.6 SHELL GEOMETRY

Geometry routines are included in the program for the following common shell types:

- Plate
- Cylinder
- Cone/Annular Plate
- Sphere
- Torus
- Ellipsoid
- Parabolid
- Hyperboloid
- Elliptic Cylinder
- Elliptic Cone

These are denoted by the parameter NSHELL = 1 through 10 as discussed in Section 6. In addition, two dummy routines (NSHELL = 11 and 12) are included that are used for shell geometries not listed above. These enable the user to provide his own geometry. The following are instructions for the generation of such routines.

A set of coordinates, the surface coordinates, are defined so that a pair of values (ξ, η) uniquely define the position of a point on the shell surface. ξ, η and the outward normal n should conform to the right-hand rule. For example, for a cylinder it is practical to choose the axial and the angular coordinates. In addition, a set of orthogonal Cartesian coordinates x, y, z is defined, and the coordinate values in this system for a point on the shall surface are expressed in terms of surface coordinates.

$$\begin{aligned} x &= f(\xi, \eta) \\ y &= g(\xi, \eta) \\ z &= h(\xi, \eta) \end{aligned} \tag{39}$$

These expressions are then used to generate the coefficients of the first and second fundamental forms [Eqs. (40,41)] of the shell surface and the normal to the surface [Eq. (42)].

$$I = A^2 d\xi^2 + 2C\, d\xi\, d\eta + B^2 d\eta^2 \tag{40}$$

$$II = D d\xi^2 + 2E\, d\xi\, d\eta + F d\eta^2 \tag{41}$$

$$n = (n_1, n_2, n_3) = \frac{1}{(A^2B^2 - C^2)^{1/2}} \Big[(g,_\xi h,_\eta - h,_\xi g,_\eta), (f,_\eta h,_\xi - h,_\eta f,_\xi), (f,_\xi g,_\eta - g,_\xi f,_\eta) \Big] \tag{42}$$

where

$$\begin{aligned} A^2 &= f_{,\xi}^2 + g_{,\xi}^2 + h_{,\xi}^2 \\ B^2 &= f_{,\eta}^2 + g_{,\eta}^2 + h_{,\eta}^2 \\ C &= f,_\xi f,_\eta + g,_\xi g,_\eta + h,_\xi h,_\eta \\ D &= n_1 \cdot f,_{\xi\xi} + n_2 \cdot g,_{\xi\xi} + n_3 \cdot h,_{\xi\xi} \\ E &= n_1 \cdot f,_{\xi\eta} + n_2 \cdot g,_{\xi\eta} + n_3 \cdot h,_{\xi\eta} \\ F &= n_1 \cdot f,_{\eta\eta} + n_2 \cdot g,_{\eta\eta} + n_3 \cdot h,_{\eta\eta} \end{aligned} \tag{43a-f}$$

The derivatives of these coefficients with respect to the surface coordinates, ξ, η, are also required. These are written

$$AX = \partial A/\partial\xi \ , \ BX = \partial B/\partial\xi \ , \ldots , \ FX = \partial F/\partial\xi$$
$$AY = \partial A/\partial\eta \ , \ BY = \partial B/\partial\eta \ , \ldots , \ FY = \partial F/\partial\eta \qquad (44)$$

If the surface coordinates ξ, η are nonorthogonal, NSHELL is set equal to 12 and the user provides a subroutine UNORTH (see Section 5.5). If the coordinates are orthogonal, the procedure is simplified since $C = E = 0$. In that case NSHELL should be set equal to 11 and the user provides a subroutine ORTH (see Section 5.4). Examples of the subroutines ORTH and UNORTH are presented below.

Consider the development of the subroutine ORTH for a paraboloidal shell with meridian defined by

$$x = ar^2 \qquad (45)$$

A cylindrical system with the axial coordinate ξ and the angular coordinate η can be chosen as surface coordinates. The Cartesian coordinates are expressed in terms of the surface coordinates:

$$x = f(\xi, \eta) = \xi$$
$$y = g(\xi, \eta) = (\xi/a)^{1/2} \sin \eta \qquad (46)$$
$$z = h(\xi, \eta) = (\xi/a)^{1/2} \cos \eta$$

Hence

$$f,_{\xi} = 1 \qquad f,_{\xi\xi} = 0$$
$$g,_{\xi} = \sin \eta/(2\sqrt{\xi a}) \qquad g,_{\xi\xi} = -\sin \eta/(4\xi\sqrt{\xi a})$$
$$h,_{\xi} = \cos \eta/(2\sqrt{\xi a}) \qquad h,_{\xi\xi} = -\cos \eta/(4\xi\sqrt{\xi a})$$

$$f,_{\eta} = 0 \qquad f,_{\eta\eta} = 0$$

$$g,_{\eta} = (\xi/a)^{1/2} \cos \qquad g,_{\eta\eta} = -(\xi/a)^{1/2} \sin \eta$$

$$h,_{\eta} = -(\xi/a)^{1/2} \sin \eta \qquad h,_{\eta\eta} = -(\xi/a)^{1/2} \cos \eta \tag{47}$$

The components of the normal are

$$n_1 = -\left(\sin \eta/(2\sqrt{a\xi}) \cdot \sin \eta \sqrt{\xi/a} + \cos \eta/(2\sqrt{a\xi}) \cdot \cos \eta \sqrt{\xi/a}\right)\frac{1}{AB} = -\frac{0.5}{a}\frac{1}{AB}$$

$$n_2 = \left(\sin \eta \cdot \sqrt{\xi/a}\right)\frac{1}{AB}$$

$$n_3 = \left(\cos \eta \cdot \sqrt{\xi/a}\right)\frac{1}{AB} \tag{48}$$

Substitution of equations (47 and 48) into (43a–f, 44) yields

$$A = \left[1 + \frac{1}{4}\frac{\sin^2 \eta}{\xi a} + \frac{1}{4}\frac{\cos^2 \eta}{\xi a}\right]^{1/2} = \sqrt{1 + 1/(4a\xi)}$$

$$B = \left[\frac{\xi}{a}\cos^2 \eta + \frac{\xi}{a}\sin^2 \eta\right]^{1/2} = \sqrt{\xi/a}$$

$$C = 0$$

$$AX = \frac{1}{2}[1 + 1/(4a\xi)]^{1/2}\frac{1}{4a}\left(-\frac{1}{\xi^2}\right) = -1/[4\xi\sqrt{a\xi(1 + 4a\xi)}] \tag{49}$$

$$BX = 1/(2\sqrt{a\xi})$$

$$AY = BY = 0 = CX = CY$$

$$D = \frac{1}{AB}\left(-\frac{1}{4}\sin^2 \eta \cdot \frac{1}{a} \cdot \xi^{1/2} \cdot \xi^{-3/2} - \frac{1}{4}\cos^2 \eta \cdot \frac{1}{a} \cdot \frac{1}{\xi}\right) = -1/(4aAB\xi)$$

$$F = \frac{1}{AB}\left(-\sin^2 \eta \cdot \frac{\xi}{a} - \cos^2 \eta \cdot \frac{\xi}{a}\right) = -\xi/(aAB) \tag{50}$$

$$DX = \frac{1}{4a}\frac{(AX \cdot B + BX \cdot A)\xi + AB}{(AB \cdot \xi)^2} \qquad FX = -\frac{1}{a}\frac{A \cdot B - (AX \cdot B + BX \cdot A)\xi}{(AB)^2}$$

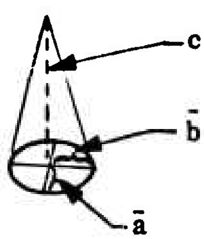
$$DY = 0 \qquad FY = 0$$

$E = 0$

Table 4 (Section 5) shows the corresponding subroutine ORTH for this example.

As an example of a shell with nonorthogonal coordinates, the development of the Subroutine UNORTH for an elliptical cone is given here.

The following sketch provides the geometry of an elliptical cone. The shell surface and its boundaries are defined by the distances $\bar{a}$, $\bar{b}$, and c. The geometrical constants occurring in the kinematic relations can most conveniently be determined if an elliptical coordinate system is used in the definition of the shell midsurface. Hence with



$$a = \bar{a}/c \quad \text{semi-major axis}$$

$$b = \bar{b}/c \quad \text{semi-minor axis} \tag{51}$$

the coordinates in a Cartesian system x, y, z can be written

$$\begin{aligned} x &= a\xi \cos \eta \\ y &= b\xi \sin \eta \\ z &= \xi \end{aligned} \tag{52}$$

The surface coordinate ξ represents the distance from the apex of the cone and η can be expressed in terms of the angular coordinate ϕ through

$$\eta = \arctan\left(\frac{a}{b}\tan\phi\right) \tag{53}$$

The coordinate lines intersect one another in an angle

$$\theta = \arccos\,[C/(AB)] \tag{54}$$

The coefficients in the first fundamental form becomes

$$\begin{aligned} A &= (1 + a^2\cos^2\eta + b^2\sin^2\eta)^{1/2} \\ C &= -\xi(a^2 - b^2)\sin\eta\cos\eta \\ B &= \xi\,(a^2\sin^2\eta + b^2\cos^2\eta)^{1/2} \end{aligned} \tag{55}$$

The derivatives of these coefficients are

$$\begin{aligned} AX &= 0 \\ CX &= C/\xi \\ BX &= B/\xi \\ AY &= -(a^2 - b^2)\sin\eta\cos\eta/A \\ CY &= -\xi(a^2 - b^2)(\cos^2\eta - \sin^2\eta) \\ BY &= \xi^2\sin\eta\cos\eta(a^2 - b^2)/B \end{aligned} \tag{56}$$

The coefficients of the second fundamental form and their derivatives are

$$\begin{aligned} D &= E = 0 \\ DX &= EX = 0 \\ DY &= EY = 0 \end{aligned}$$

$$F = + ab\xi^2/H$$

$$FX = F/\xi$$

$$FY = -ab\xi\,(\xi/H)^3 \sin\eta \cos\eta\,(a^2 - b^2) \tag{57}$$

where

$$H = \xi\,(a^2 \sin^2\eta + b^2 \cos^2\eta + a^2b^2)^{1/2} \tag{58}$$

Table 5 (Section 5) shows the subroutine UNORTH for an elliptical cone. Note: in this example the location of columns in the output will be in η coordinate rather than in angular coordinate ϕ.

3.7 LOADING

The program will accept input defining two independent load systems. During one run, the loads in the two systems may both be proportionally increased, or either one can be restrained by use of the "Maximum Load" input. The collapse analysis for a shell may consist of several runs, the restarting capability is an essential feature of the computer program. This restarting capability also allows the user to change the relation between the increments of the two load systems at any given level of loading. This makes it (for example) possible to apply a fixed external pressure and, in subsequent runs, to keep the pressure constant while an axial shortening is applied until collapse occurs. The loading can be either a prescribed load or a prescribed displacement at any of the grid points. In the bifurcation analysis, the critical loading is defined as the initial load for system B + eigenvalue times the initial load for system A. Consequently if we want to find the critical axial load of a shell with a fixed internal pressure we can represent the internal pressure by load system B and the axial compression by load system A.

The load systems can contain uniform surface tractions or line loads in any direction. The line loads can be applied in the form of a specified load or a specified displacement along any of the boundary lines. Point loads or w displacements can be defined at any

grid point. In addition, the user has the option to add a subroutine which specifies nonuniform pressure or line loads as functions of the shell coordinates. Such nonuniform loads are internally converted into point forces at the grid points.

3.8 CONSTITUTIVE RELATIONS

The constitutive equations are here used in the following form.

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_{12} \\ M_1 \\ M_2 \\ M_{12} \end{Bmatrix} = [C_{ij}] \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \\ \kappa_1 \\ \kappa_2 \\ 2\kappa_{12} \end{Bmatrix}$$

The stiffness coefficients, C_{ij}, can be computed from the elastic and geometric properties of the shell wall. Special subroutines in which these properties are read and the stiffness coefficients computed are provided for

1. Monocoque orthotropic shells
2. Shells with skew stiffeners (waffle pattern)
3. Fiber reinforced [illegible]
4. Layered [illegible] (orthotropic layers)
5. Shells with corrugated skin
6. Shells with one corrugated and one smooth skin
7. Shells in which the elastic properties vary gradually through the thickness

In addition, a subroutine is provided in which the stiffness coefficients are modified to include the effects of "smeared" stiffeners.

For generality, an option is provided in which the user writes his own wall property routine (WALL). Instructions are given here for the derivation of such subroutines;

subsequent text provides an example. This option provides the only method for introducing variable wall properties (except variable elastic properties in monocoque shells; see input pr paration section under card M-1 for IWALL = 8 or 9). The derivation of the user-written wall property routine (WALL), has been facilitated because standard subroutines for shell types listed above can be called from WALL.

For example, in his routine the user can define such properties as moduli, shell thicknesses, and stiffener data as functions of the surface coordinate, and call the appropriate shell wall subroutine which will return the corresponding stiffness coefficients. The user-written subroutine will automatically be called whenever stiffness coefficients are needed if the parameter IWALL (M-1 card) is set equal to one. More than one of the special routines may be called from the user written routine if the shell wall is of different type in different areas.

For a case in which the shell wall is of a type not included among the standard types, stiffness coefficients (C_{ij} matrix) may be read directly in WALL. The coefficients may be derived as described in the following. The strain energy, U, is expressed in terms of strains and changes of curvature. If a subscript following a comma indicates differentiation with respect to one of the strains and changes of curvature such that for $i = 1,2,3,4,5,6$ derivatives are taken with respect to $\epsilon_1, \epsilon_2, \gamma_{12}, \kappa_1, \kappa_2, 2\kappa_{12}$, respectively, then

$$C_{ij} = U_{,ij} \tag{59}$$

Stiffness coefficients for the standard wall types are given in Ref. 14 which also gives as an example the detailed derivation for one case, the fiber reinforced shell. As a somewhat simpler example we will demonstrate here, how the coefficients are obtained for a shell with rectangular stiffeners in one direction. It is assumed that the effects of torsional stiffness and resistance to rotation of the stiffener can be neglected.

If

$$\bar{\epsilon}_x, \bar{\epsilon}_y, \text{ and } \bar{\gamma}_{xy}$$

denote strains at the reference surface (here the midsurface of the skin) and

$$\kappa_x, \kappa_y, \text{ and } \kappa_{xy}$$

are the changes of curvature, then the strains at any point off the reference surface can be obtained from

$$\epsilon_x = \bar{\epsilon}_x + \kappa_x Z \qquad \text{in skin and stringer}$$

$$\epsilon_y = \begin{cases} \bar{\epsilon}_y + \kappa_y Z & \text{in skin} \\ 0 & \text{in stringer} \end{cases}$$

$$\gamma_{xy} = \begin{cases} \bar{\gamma}_{xy} + 2\kappa_{xy} Z & \text{in skin} \\ \text{irrelevant} & \text{in stringer} \end{cases}$$

The geometric properties of the shell wall are shown in Fig. 3-4.

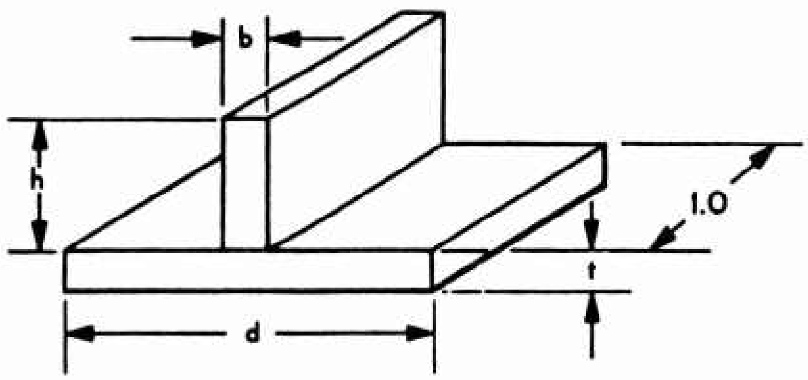


Fig. 3-4

The strain energy per unit area of the shell wall is

$$U = \frac{1}{2d} \int_V \left(\sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy} \gamma_{xy} \right) dv \tag{60}$$

The contribution from the skin is

$$U = \frac{Et}{2(1 - \nu^2)} \left(\bar{\epsilon}_x^2 + \bar{\epsilon}_y^2 + 2\nu \bar{\epsilon}_x \bar{\epsilon}_y + \frac{1 - \nu}{2} \bar{\gamma}_{xy}^2 \right)$$

$$+ \frac{Et^3}{24(1 - \nu^2)} \left[\kappa_x^2 + \kappa_y^2 + 2\nu \kappa_x \kappa_y + 2(1 - \nu)\kappa_{xy}^2 \right] \tag{61}$$

For the stringer we have

$$U_{STR} = \frac{Eb}{2d} \int_{t/2}^{h+t/2} (\bar{\epsilon}_x + Z\kappa_x)^2 \, dZ$$

$$= \frac{Eb}{2d} \left[h\bar{\epsilon}_x^2 + (h^2 + ht)\bar{\epsilon}_x \kappa_x + \left(\frac{h^3}{3} + \frac{h^2 t}{2} + \frac{ht^2}{4} \right) \kappa_x^2 \right] \tag{62}$$

or with

$A = bh$ = stringer area

$I = bh^3/12$ = stringer moment of inertia

$e = (h + t)/2$ = stringer eccentricity

$$U_{STR} = \frac{E}{2d} \left[A\bar{\epsilon}_x^2 + 2Ae\bar{\epsilon}_x \kappa_x + (I + Ae^2)\kappa_x^2 \right] \tag{63}$$

Then by use of Eq. (59) we find

$$C_{11} = \frac{Et}{1 - \nu^2} + \frac{EA}{d} \qquad C_{22} = \frac{Et}{1 - \nu^2}$$

$$C_{12} = \frac{\nu Et}{1 - \nu^2} \qquad C_{33} = \frac{Et}{2(1 + \nu)}$$

$$C_{14} = \frac{EAe}{d}$$

$$C_{44} = \frac{Et^3}{12(1 - \nu^2)} + \frac{E(I + Ae^2)}{d} \qquad C_{55} = \frac{Et^3}{12(1 - \nu^2)}$$

$$C_{45} = \frac{\nu Et^3}{12(1 - \nu^2)} \qquad C_{66} = \frac{Et^3}{24(1 + \nu)} \tag{64}$$

If a user written shell wall property routine is used,certain rules must be followed. Thus the list of variables under the subroutine name (WALL) is (X,Y,CCC) which represent the shell surface coordinates and stiffness coefficients. The X and Y dimensions must correspond to those shown in Section 5, page 5-1.

If any of the other routines are called from WALL the list in the call statement must be (N). Any number of the routines listed in Table 7 may be called from WALL. The table also shows the required common statements corresponding to each of the subroutines. As there are no cards to be read in the routines called from WALL it is necessary to provide in WALL the size of the quantities in these common statements. They are defined on the cards M-2B through N-4B in the input description. If smeared stiffeners data are provided in WALL the user must CALL STIFF in subroutine WALL to generate the stiffeners coefficients.

As an example we will consider here a shell which is composed of a spherical segment and a cylindrical part as shown in Fig. 3-5.

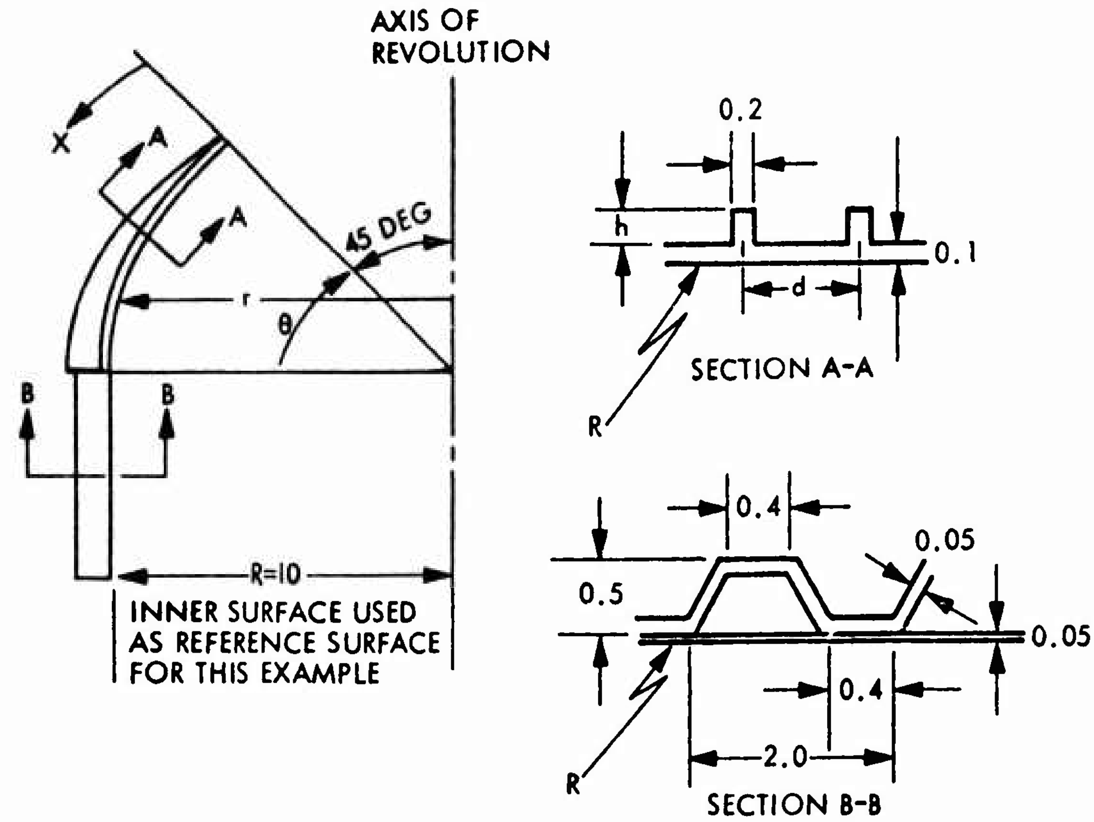


Fig. 3-5

The user written geometry which is needed for this case we will assume uses as X coordinate the arc-length from the upper edge. Hence the local radius

$$r = 10 \cdot \cos\left(\frac{\pi}{4} - \frac{X}{10}\right)$$

if there are 40 stringers around the circumference we have

$$d = \frac{2\pi \cdot r}{40} = \frac{\pi \cdot r}{20}$$

We assume that it is specified that the stringer height varies linearly from zero at the shell edge to 0.6 inch at the interface between the spherical and cylindrical shells.

$$h = \frac{X \cdot 0.6}{(\pi/4)} = \frac{2.4 \cdot X}{\pi}$$

The material for both parts of the shell is isotropic with $E = 10^7$ and $\nu = 0.3$. The routine WALL corresponding to this shell is listed as an example in Table 5-7.

Section 4
STRATEGY

The preceding paragraphs have described the scope and limitations of the STAGS computer program. The following text discusses the choice of certain control parameters; and an effort is made to convey to the user some of the experience that has been gained through extensive program use.

There are essentially two areas within which the user is required to use his own judgment. The first of these is the modeling of a structure so that it becomes amenable for analysis. This will include the choice of a able grid. The second area is the choice of certain parameters, such as the initial load step and the convergence criterion, which will govern the flow of computations ne nonlinear analysis.

The modeling of the structure is generally consider be outside of the scope of this manual. It is assumed that the user has decided abo the shell geometry and material properties, about boundary conditions and loading, and about what to include in terms of stiffeners and cutouts. The next step would be to determine which type of analysis to apply. If the interest is in the stress distribution for loads which are known to be small in comparison to the collapse load of the shell a purely linear analysis can be used. If it can be safely assumed that changes in geometry or stress distribution are negligible at loads only slightly below collapse, the branch for bifurcation buckling can be used for establishment of the stability limit.

In case there is some uncertainty about this, the following procedure might help the user to avoid an expensive complete nonlinear analysis. The design load is used as both initial and maximum load. The linear analysis then will be obtained, and an attempt will be made at solution of the nonlinear equations at this load step. If the linear analysis represents a good approximation, convergence will be obtained within

a few iterations and, as no additional factoring is required, this will add very little to the computer time. If convergence is not obtained, this will serve as a warning that a nonlinear analysis may be needed. It is recommended, therefore, that the nonlinear option be used unless similar cases previously analyzed clearly indicate that a linear analysis is satisfactory. Also, note that a slight increase in load beyond the design load may lead to a considerable increase in the influence of nonlinear terms. Such behavior would be revealed by a bifurcation analysis. If results from the nonlinear analysis at the design load differ little from results from a linear analysis and the bifurcation load is well above the design load, shell collapse is not a problem. In choosing the type of analysis, it may be useful for the program user to know that a nonlinear analysis, if it converges at the first step only requires about 25 percent more computer time than a linear analysis, and that a bifurcation buckling analysis could take from two to three times as much computer time as a linear analysis, when multiple eigenvalues exist in the neighborhood of the lowest eigenvalue.

If the shell is subjected to such load as to make instability a possibility, and if the shell dimensions are not determined by other – nonstructural – requirements, efficient design can be achieved only if a collapse analysis is performed. In this case, the bifurcation buckling analysis should be used only if the analysis of similar cases clearly indicates its adequacy.

The next step is to determine the size of the grid. For economy, it is necessary to take advantage, whenever possible, of the capability of using nets with variable spacing. If similar cases have not been run before, it is essential that a convergence study be made. Often this may be done by use of a linear analysis or a bifurcation buckling analysis. Note, however, that sometimes a good estimate of the collapse load can be obtained with a more coarse grid than is required for an accurate estimate of prebuckling stress distribution. Also, sometimes, the coarseness of a net which is satisfactory for linear analysis, will result in spurious buckling modes.

The user can reasonably well determine the computer time for a linear stress analysis or bifurcation buckling analysis from Fig. 4-1. For a nonlinear analysis, it is difficult to predict how many refactorings are needed and how big the local steps may be. Depending upon how drastic the changes in geometry or stress distribution are in the

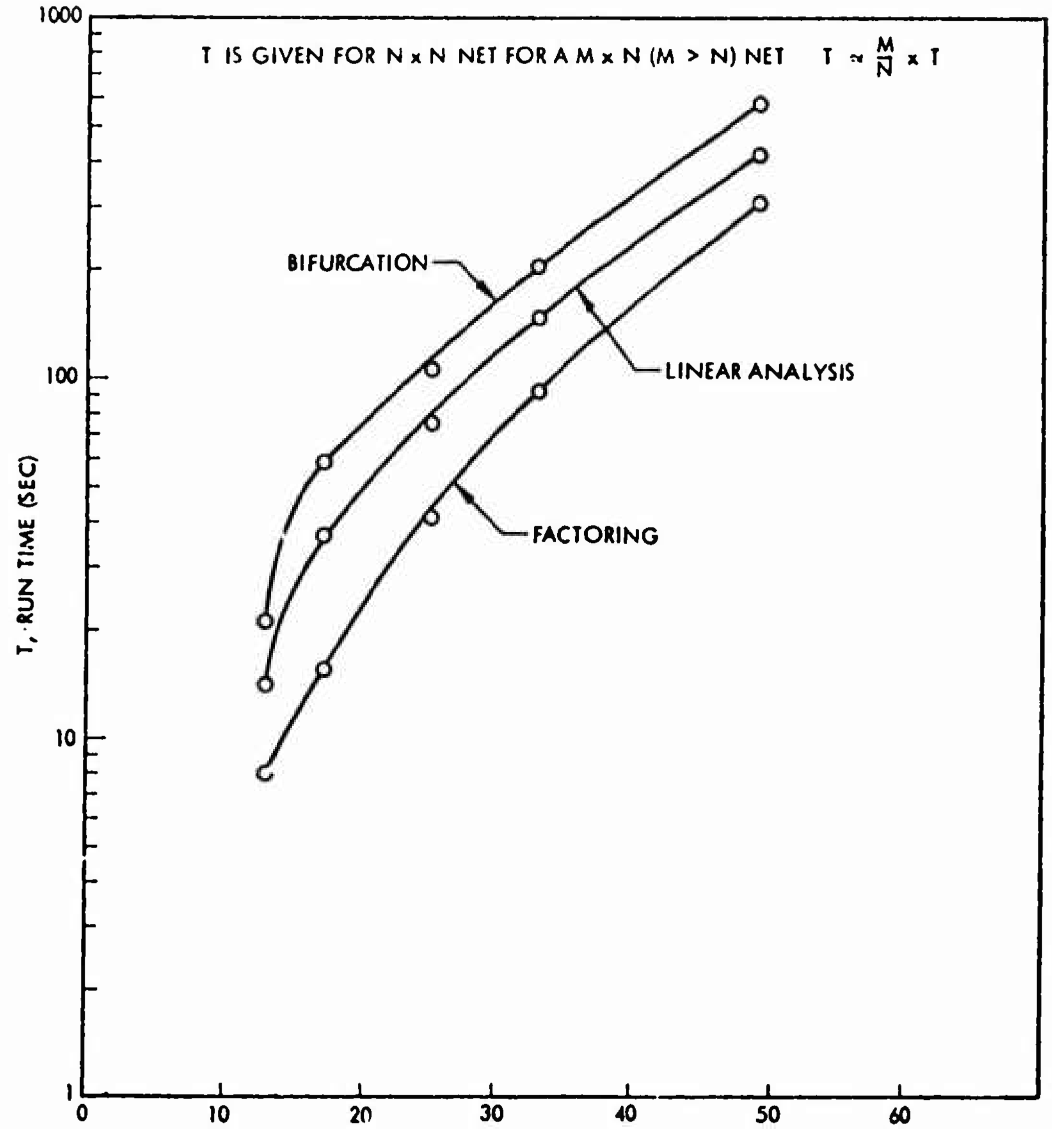


Fig. 4-1 CDC 6600 Computer Time for Bifurcation and Linear Stress Analysis

critical range, the computer time for a collapse analysis varies from, for example, 10 to 100 times the time for a linear analysis.

Once the structural model (including the grid) has been chosen, the linear analysis is straightforward; for a nonlinear analysis additional decisions must be made. The user must choose the initial load step and other control parameters and also may elect to override built-in values for the iteration convergence criterion and the over- or under-relaxation factor. An effort has been made to make automatic as much as possible of the computation strategy. Thus, for example, if convergence is slow, an over- or under-relaxation factor is automatically applied depending on whether the convergence is uniform or oscillating. The user generally leaves blank fields for convergence parameter and relaxation factor. This means that the convergence parameter is set to 10^{-4}, and the relaxation parameter is chosen as indicated above. The user may loosen this convergence criterion if it is indicated from the results that round-off errors make convergence difficult or even impossible. The criterion also may be tightened as discussed below. The relaxation factor should be determined by the user only if the user has considerable experience in the use of the program. If the relaxation factor is given any value on the input card (including 1.0), this factor will be used throughout the analysis.

The linear solution is always computed and used as a first estimate for the unknowns. Later, estimates are obtained through extrapolation (quadratic when two or more previous solutions are available). Therefore, the initial load must be chosen so that the solution for this load level is not too far from the linear solution. A bifurcation analysis to guide in the choice of initial stepsize, although expensive, is possible if there is no other basis for a reasonable guess. If convergence is obtained easily within one or two iterations, the load-step will be automatically increased by a factor of 1.5. This increase will be repeated any number of times, and thus the penalty for a poor initial load step guess will be alleviated. This feature in the program also is useful for many cases in which a proper guess is made for starting values. For example, for a shell of revolution with nonuniform load, convergence is usually slow at the first couple of load steps; after previous solutions are available for extrapolation, the convergence is faster and the load step can be increased.

The choice of initial load and of initial size of the load step for maximum efficiency is case dependent. A user who is experienced in nonlinear analysis can choose these parameters to suit his particular case. To a user who has no experience on which to base such a decision, one might recommend that initial load and initial step be chosen at about one tenth of the anticipated collapse load. If the structure is expected to behave almost linearly until the critical load is closely approached, the initial load may be large and the initial step much smaller.

If convergence is not obtained in the first load step, it is probably because the initial loads are too big, and the nonlinear solution differs too much from the linear solution. However, there are other possibilities such as simple mistakes in input quantities. The user also is advised to check whether the boundary conditions allow rigid body motions – translations, rotations, or combinations of these. It also is possible that the boundary conditions allow deformation in a mode in which the shell is very weak, e.g. an inextensional deformation mode. For such cases, it is possible that the system is so ill conditioned that the problem cannot be solved with single precision accuracy. If this deformation mode is not essential to the solution of the problem, it can be restrained and subsequently a successful analysis can be executed. The cases in which such difficulties occur are quite rare.

If the convergence was not obtained because the initial load step was too large, then this step must be reduced. If the iteration diverges, the load may be cut by as much as a factor of 10, whereas a lesser reduction may be successful if the iteration converges too slowly. After initial convergence has been obtained, the program continues with the input load step until a load level is reached at which the program fails to converge within prescribed limits. The program then either cuts the load increment in half or makes a standard Newton iteration step by recomputing $L'(X_m)$ where X_m is the displacement vector solution obtained for the previous load step. The choice between these two alternatives is based on strategy parameters input by the user:

- ICUT – Total number of times load increment may be cut in half
- INEWT – Number of times the matrix $L'(X_m)$ may be recomputed
- ISTRAT – Number of times load increment is cut between recomputation of $L'(X_m)$.

Thus, if ISTRAT = 1, the program alternates between cutting the load increment and recomputing the matrix when convergence difficulties are encountered. When the program has already cut the step size ICUT times, additional "Newton" steps (recomputation and factorization of $L'(X_m)$) are permitted until the total exceeds INEWT. Of course the reverse situation also may occur in which the program is only permitted to cut the step size but not to take additional "Newton" steps. If neither strategy is allowed, the program stops as soon as convergence difficulties arise.

In view of the highly unpredictable nature of nonlinear behavior, it is very difficult to prescribe the best values for the strategy parameters in advance. As an initial choice for an unfamiliar type of shell, the values

ICUT=2
INEWT=2
ISTRAT=1

are suggested. By observing the convergence behavior in previous computer runs, a more effective set of values may be selected for continuation. Thus, if for example, the last five load steps converge in one iteration each, a larger load increment may be desirable. Conversely, as a collapse load is approached, it will be necessary to cut the load increment frequently (e.g., ISTRAT=2 or ISTRAT=3). Sometimes a shell exhibits substantial redistribution of stresses at loads well below the collapse load. In such regions, the most economical strategy may be to take "Newton" steps (which take into account the stress redistribution) more frequently than cutting the step size. Such a strategy could be achieved by

ICUT=1
INEWT=6
ISTRAT=1

In general, it has been found advisable to restrict each computer run to 5 to 10 times the computer time for factoring so that the convergence behavior can guide the selection of strategy.

The STAGS program is, of course, not an appropriate tool for analysis of the buckling or collapse of shells of revolution under axisymmetric loading. For such problems, simpler tools are available, such as the BOSOR4 program (Ref. 15). Axisymmetric cases have been analyzed for verification of the validity of the STAGS program. Theoretically, as the critical load is reached, the round-off errors should trigger a deformation in the buckling mode. In practical application, it is generally found that in the early stage of buckling the amplitude of the buckling mode is so small in comparison to the prebuckling displacement that its growth, although in a relative sense large, will not violate the specified convergence criterion. This difficulty is avoided by the specification of initial imperfections in the shell geometry which are small enough not to appreciably affect the buckling load but large enough to trigger the new deformation pattern. The program, therefore, has been equipped with an option to add a subroutine which describes an initial lateral displacement pattern.

It has been found during use of the program that the imperfections may be useful as triggers also in other cases than those with perfect axial symmetry. For example, in the analysis of an elliptical cylinder with an aspect ratio of 1.5, it was found that without trigger it was necessary to use a very severe convergence criterion ($\epsilon = 10^{-5}$) but if a small imperfection is added, the same collapse load may be computed in about half the run time with a less severe convergence criterion ($\epsilon = 10^{-3}$). If the elliptical cylinder has a significantly smaller aspect ratio, it is likely that the amplitude of the buckling pattern which is present in the prebuckling displacement is too small to act as a trigger. In this case, the computation of a collapse load must include the use of a small imperfection. The choice of imperfection mode may often be aided by knowledge of the bifurcation buckling mode.

If difficulties like these do occur, they will be discovered when attempted refactoring at a load level above the collapse load leads to a coefficient matrix for the linear system which is not positive definite. In such a case, the user must either sharpen his convergence criterion or introduce an imperfection.

The run may often be saved if a new run is restarted from an earlier solution; i.e., ISTART is chosen to be 1 or 2 rather than 3, which is the value chosen under normal conditions. There also may be other reasons to suspect that an inaccurate solution has been accepted in which case restart from file 1 or file 2 on the data tape is advisable.

Note that through use of additional analysis with various degree of imperfections, the user of the program can get some notion of the degree of imperfection sensitivity of the collapse load.

The input card with strategy parameters (P-1B card) also includes a parameter ISEC. Occasionally, during computations, a check is made of whether the elapsed computer time exceeds ISEC; in this case, intermediate results are saved on data tape. ISEC should first be chosen to be a minute or so less than the time estimate at which the operator aborts the run. Before each refactoring, the program also checks that sufficient time is available to make refactoring meaningful. To refactor at the end of a run would be wasteful because a restarted run begins with factoring.

In the bifurcation analysis, inverse power iteration is used to obtain the critical value closest (in absolute value) to the initial shift point. The rate of convergence to the critical mode may be very slow unless a shift of the eigenvalue spectrum is used. If the parameter ISHIFT is set greater than zero, the program automatically performs up to a maximum of ISHIFT eigenvalue shifts to expedite convergence. It is proposed that if the user has no special reason to do otherwise, ISHIFT is set to 2 and ITERAT (the maximum number of iterations between shifts) is set to 20.

Whenever the critical load is reasonably well known, it is probably desirable to use a value somewhat below the expected buckling load as initial shift. In general, it should be noted that increased convergence rates are obtained at the cost of a complete matrix factorization for each eigenvalue shift. An initial shift may sometimes be necessary to obtain a physically meaningful critical value which does not correspond to the lowest mathematical eigenvalue. For example, when a load system results in tension somewhere in the structure, there will usually be negative eigenvalues which may not be of interest. Also, it may not be convenient to eliminate rigid body motion by means of boundary conditions in which case there will be eigenvalues approximately or exactly zero. In these cases, the physically meaningful buckling load can be obtained by an initial shift which is sufficiently close to the desired critical value.

Section 5

USER-WRITTEN SUBROUTINES

To extend the applicability of the STAGS computer program, the option of several user-written subroutines was provided. These subroutines make it possible for the user to communicate to the system functional relationships which would be difficult, if not impossible, to define through regular data card input. Do not read input cards in any of these routines.

Some of the information defined in the user-written subroutines may, in effect, override data read in on regular input cards, but none of the user-written routines suppresses the reading of any of these cards.

Instructions and examples of user-written subroutines are given here except for the geometry routines (ORTH and UNORTH) and routine WALL which are discussed in Section 3.

The coordinates X and Y used in the user-written subroutines must correspond to the coordinates used for the description of the shell geometry (NSHELL) as follows:

NSHELL	GEOMETRY	X	Y
1	Cylinder	Length	Degrees
2	Cone/Annular Plate	Length	Degrees
3	Plate	Length	Length
4	Sphere	Degrees	Degrees
5	Paraboloid	Length	Degrees
6	Elliptic Cylinder	Length	Degrees
7	Ellipsoid	Degrees	Degrees
8	Torus	Degrees	Degrees
9	Hyperboloid	Degrees	Degrees
10	Elliptic Cone	Length	Degrees
11	ORTH	As specified by user in geometry routine	
12	UNORTH		

If trigonometric terms are used in a user-written subroutine, the arguments must be in radians. That is, if X and/or Y are in degrees, they must be converted to radians in the user-written subroutine (see Section 5.1 and Table 1 for example).

The portion of the user-written subroutine that must be added to the program file is clearly marked with an asterisk (✱) for particular examples, in Tables 1 to 7. The subroutines names, dimensions, comments, common statements, and the return and end cards are part of the permanent program file and appear in it in consecutive order.

5.1 FUNCTION WIMP (K,X,Y)

This routine defines initial imperfection, if any, of the shell surface.

The computer program uses only the first derivatives of the imperfection with respect to the two space variables. If, when the subroutine is entered, the parameter in the list (K) is equal to 2, the derivative with respect to Y is requested (Y is zero on boundary line 4). Otherwise, the subroutine should return only the derivative with respect to X (X is zero on boundary line 1).

Table 1 gives an example using this routine for a cylinder with L = 10 and W_o = 0.00001 sin $(X\pi/2L)$ cos (6Y).

5.2 SUBROUTINE USRLD (X,Y,NROW,NCOL)

This subroutine serves to define a functional relationship between the external loading and the X, Y mesh coordinates and is called only if LFLG = 1 on the L-1 type input card. Additional loads can be defined by use of the L-2 cards.

X(I)	=	X coordinate of mesh point I
Y(I)	=	Y coordinate of mesh point I
NROW	=	Number of rows
NCOL	=	Number of columns

In the process of coding the USRLD subroutine, the user has to define the external load P at any or all mesh points and then issue a call statement

CALL FORCE (L, M, N, P, I)

where

- L = Row number of mesh point where P is acting
- M = Column number of mesh point where P is acting
- N = Direction of P
  - 1 – Normal (Z)
  - 2 – Tangential (Y)
  - 3 – Tangential (X)
- P = External load
- I = Load type
  - -1 – Displacement
  - 1 – Point force
  - 2 – Line load along rows
  - 3 – Line load along columns
  - 4 – Pressure test
  - 5 – Line pressure load (uniform pressure only)

The positive load is applied in the direction of positive displacement.

Table 2 provides an example using this routine for internal pressure that varies along the Y coordinate according to $P = 10 [\cos(2Y) + 1]$.

5.3 SUBROUTINE MATER (X, Y, IP, TDEG, EX, EY, U, G, A1, A2)

This subroutine defines the temperature and wall properties at every mesh point and point through the thickness in the shell and is called by the program only if IWALL = 8 on the M-1 type input card.

- X = X coordinate of mesh point
- Y = Y coordinate of mesh point
- IP = Number of points across the wall (numbered inner to outer)
- TDEG = Wall temperature

EX	=	Modulus of elasticity in X direction
EY	=	Modulus of elasticity in Y direction
U	=	Poisson's ratio (μ_{xy})
G	=	Shear modulus
A1	=	Coefficient of thermal expansion in X direction
A2	=	Coefficient of thermal expansion in Y direction

An example using this routine for material properties as function of a parameter C is given in Table 3, where

$$C = 1.0 - 0.4 \cos(Y) \quad \text{for } Y < 90$$

$$C = 1.0 \quad \text{for } Y \geq 90$$

5.4 SUBROUTINE ORTH (PROP,X,Y)

This subroutine defines shell geometries described by a set of orthogonal surface coordinate lines. It must be provided by the program user when the parameter NSHELL (see Section 6, Card G-2 is set equal to 11. If a 3D plot of geometry (NCHK.GT.0) is requested, the user must provide the orthogonal cartesian coordinates XG, YG, and ZG as shown in Table 4.

An example of a paraboloid is described in Section 3.6 and the resulting subroutine shown in Table 4.

5.5 SUBROUTINE UNORTH (PROP, X,Y)

This subroutine defines shell geometries described by a set of nonorthogonal surface coordinate lines. It must be provided by the program user when the parameter NSHELL (see Section 6, Card G-2) is set equal to 12, and can be used only in conjunction of IWALL equal 1 or 2. (IWALL = 1 requires user written subroutine for shell wall properties.) If a 3D plot of geometry (NCHK. GT. 0) is requested, the user must provide the orthogonal cartesian coordinates, XG, YG, and ZG as shown in Table 5.

Example of an elliptical cone is described in Section 3.6 and the resulting subroutine is shown in Table 5.

5.6 SUBROUTINE TEMP (X, Y, T, AP1, AP2)

The user has the option of entering the thermal loading in conjunction with load Pattern A by means of this subroutine. Subroutine TEMP is called for each mesh point of the shell and normally returns a temperature value of zero unless the user specifies otherwise. X and Y are the shell coordinates of the surface. AP1 and AP2 are the coefficient of thermal expansion in X and Y direction, respectively.

Table 6 shows an example of the use of this routine for temperature variation given by $T \doteq 50\,[1 - 0.4\cos(Y)]$ where Y is given in degrees.

5.7 SUBROUTINE WALL (X,Y,CCC)

Subroutine WALL makes it possible for the user to vary the stiffness matrix or the material properties which influence the calculation of the stiffness matrix at various mesh-points. The common variables related to the various types of wall constructions are available by means of FORTRAN COMMON statements.

The subroutine is called at each mesh-point if IWALL is set to 1 on the M-1 type input card. The user may calculate the CCC stiffness matrix (6 × 6) directly or just set the appropriate wall properties to the desired value and call subroutine CFB(N) to perform the stiffness matrix computations according to wall construction type N (see Input description for the available wall construction options).

For a simple example, see the description in Section 3.8 and resulting subroutine in Table 7.

Table 1

FUNCTION WIMP

```
      FUNCTION WIMP(K,X,Y)
      WIMP = 0.0
      P = 0.00001
      PI2 = 3.14159 / 20.0
      Y1 = Y * 3.14159 / 180.0
      IF (K .EQ. 2)  GO TO 20
      WIMP = P * PI2 * COS(PI2 * X) * COS(6.0 * Y1)
      RETURN
   20 WIMP = -P * 6.0 * SIN(PI2 * X) * SIN(6.0 * Y1)
      RETURN
      END
```

Table 2

SUBROUTINE USRLD

```
      SUBROUTINE USRLD (X, Y, NROW, NCOL)
      DIMENSION  X(NROW), Y(NCOL)
      DO 10  L = 1, NROW
      DO 10  M = 1, NCOL
      T = Y(M) * 3.14159 / 180.0
      P = 10.0 * (COS(2.0 * T) + 1.0)
   10 CALL FORCE (L, M, 1, P, 4)
      RETURN
      END
```

Table 3

SUBROUTINE MATER

```
      SUBROUTINE MATER (X, Y, IP, TDEG, EX, EY, U, G, A1, A2)
      DIMENSION TDEG(IP),EX(IP),EY(IP),U(IP),G(IP),A1(IP),A2(IP)
      COMMON /OFST/ TO,Z
C     TO AND Z MUST BE SET BY THE USER
C     TO = TOTAL THICKNESS OF SHELL (TO=AT)
C     Z = DISTANCE FROM REFERENCE SURFACE TO MIDSURFACE OF SHELL WALL
      TO = 0.1
      Z = 0.
      C = 1.0
      IF (Y .LT. 90.0) C = 1.0 - 0.4 * COS(Y * 3.14159 / 180.0)
      DO 1  L = 1, IP
      TDEG(L) = 0.0
      EX(L) = C * 1000000.0
      EY(L) = 0.4 * C * 1000000.0
      U(L) = 0.1
      G(L) = 0.2 * C * 1000000.0
      A1(L) = 0.0
      A2(L) = 0.0
    1 CONTINUE
      RETURN
      END
```

Table 4

SUBROUTINE ORTH

```
      SUBROUTINE ORTH (PROP, X, Y)
      DIMENSION PROP(3)
C     PROP(1) = X COORDINATE OF BOUNDARY LINE 1
C     PROP(2) = X COORDINATE OF BOUNDARY LINE 3
C     PROP(3) = Y COORDINATE OF BOUNDARY LINE 4
C     PROP(4) = Y COORDINATE OF BOUNDARY LINE 2
      COMMON / FQA / NSHELL, A, B, AX, AY, BX, BY, XG, YG, ZG
      COMMON / FQB / C, CX, CY, D, DX, DY, E, EX, EY, F, FX, FY
C     ***********************************************************
C                            PARABOLOID
C     ***********************************************************
C     X = S * Y * Y  EQUATION OF PARABOLIC MERIDIAN
C     S=1.0/(4.0*R) WHERE R IS THE DISTANCE FROM VERTEX TO FOCUS
C     PROP(5) = R  INPUT ON CARD G-2
C     PROP(3) AND PROP(4) ARE IN RADIANS FOR THIS EXAMPLE
      S = 1.0 / (4.0 * PROP(5))
      X = X + PROP(1)
      Y = Y + PROP(3)
      XG = X
      YG = SQRT(X/S) * SIN(Y)
      ZG = SQRT(X/S) * COS(Y)
      T = 0.25 / (X * S)
*     A = SQRT(1.0 + T)
      B = SQRT(X / S)
      A1 = SQRT(S * X * (1.0 + 4.0 * S * X ))
      AX = -0.25 / (X * A1)
      BX = 0.5 / SQRT(S * X)
      D = -T / ( A * B)
      F = -X / (S * A * B)
      DX = ((AX * B + BX * A) * X + A * B) / (4.0 * S * A * A *
     1        B * B * X * X)
      FX = -(A * B - (AX * B + BX * A) * X)/(S * A * A * B * B)
      RETURN
      END
```

Table 5

SUBROUTINE UNORTH

```
      SUBROUTINE UNORTH (PROP, X, Y)
      DIMENSION  PROP(3)
C     PROP(1) = X  COORDINATE OF BOUNDARY LINE 1
C     PROP(2) = X  COORDINATE OF BOUNDARY LINE 3
C     PROP(3) = Y  COORDINATE OF BOUNDARY LINE 4
C     PROP(4) = Y  COORDINATE OF BOUNDARY LINE 2
      COMMON / FQA / NSHELL, A, B, AX, AY, BX, BY, XG, YG, ZG
      COMMON / FQB / C, CX, CY, D, DX, DY, E, EX, EY, F, FX, FY
C     ***********************************************************
C                        ELLIPTICAL CONE
C     ***********************************************************
C     X = W * S * COS(T), Y = V * S * SIN(T), Z = S
C     W = PROP(5), SEMI-MAJOR AXIS WHEN S = 1, INPUT ON CARD G-2
C     V = PROP(6), SEMI-MINOR AXIS WHEN S = 1, INPUT ON CARD G-2
C     T IS ANGLE IN ELLIPTICAL COORDINATES, RADIANS
C     PROP(3) AND PROP(4) ARE IN RADIANS FOR THIS EXAMPLE
C
      X = X + PROP(1)
      Y1 =  Y + PROP(3)
      XG = W * X * COS(Y1)
      YG = V * X * SIN(Y1)
      ZG = X
      T1 = ATAN((PROP(5) / PROP(6)) * TAN(Y1))
      W2 = PROP(5) * PROP(5)
      V2 = PROP(6) * PROP(6)
      WV = PROP(5) * PROP(6)
      SH = SIN(T1)
      CH = COS(T1)
 *    SH2 = SH * SH
      CH2 = CH * CH
      A = SQRT(1.0 + W2 * CH2 + V2 * SH2)
      B = X * SQRT(W2 * SH2 + V2 * CH2)
      C = -X * (W2 - V2) * SH * CH
      AY = -(W2 - V2) * SH * CH / A
      BX = B / X
      BY = X * X * SH * CH * (W2 - V2) / B
      CX = C / X
      CY = -X * (W2 - V2) * (CH2 - SH2)
      H = X * SQRT(W2 * SH2 + V2 * CH2 + W2 * V2)
      F = WV * X * X / H
      FX = F / X
      FY = -WV * X * SH * CH * (W2 - V2) * (X / H)**3
      RETURN
      END
```

Table 6

SUBROUTINE TEMP

```
      SUBROUTINE TEMP (X,Y,T,AP1,AP2)
      T=0.0
      T=50.*(1.-.4*COS(Y*3.14159/180.))
      AP1=.00001
      AP2=.00001
      RETURN
      END
```

Table 7

SUBROUTINE WALL

```
      SUBROUTINE WALL (X, Y, CCC)
      DIMENSION  CCC(6,6)
      COMMON / OFST / TO, Z
C
C     Z = DISTANCE FROM REFERENCE SURFACE TO MIDSURFACE OF SHELL
C         WALL
C
C     TO = TOTAL THICKNESS OF SHELL WALL
C
C
C                  C F B 2
      COMMON / MONO / AT, EX1, XN1, EY1, G
C
C
C                  C F B 3
      COMMON / SKEW / E3, U3, T3, TH, A, B, H3, AK3
C
C
C                  C F B 4
      COMMON /FIBR/ EF,EM,UF,UM,LAYERS,TT(20),XX(20),BE(20),O(20)
C
C
C                  C F B 5
      COMMON /LAYD1/ TL(20),EX5(20),EY5(20),UXY(20),G5(20),LAYS
C
C
C                  C F B 6
      COMMON / CORR / CT6, E6, U6, CC6, CM6, CD6, CB6
C
C
C                  C F B 7
      COMMON /CORS/ CT7,E7,U7,CC7,CM7,CD7,CB7,ES,US,TS,PHI,ANC

C
C
C                 OPTION A
C        GENERATES A 6*6 MATRIX CCC ACCORDING TO SHELL WALL
C     CONSTRUCTION. SET Z AND COMMON VARIABLES FOR CFBN. THEN
C     CALL CFB(N).
C
C
C                 OPTION B
C     SET Z, TO, AND THE 6*6 MATRIX CCC.
C
C
```

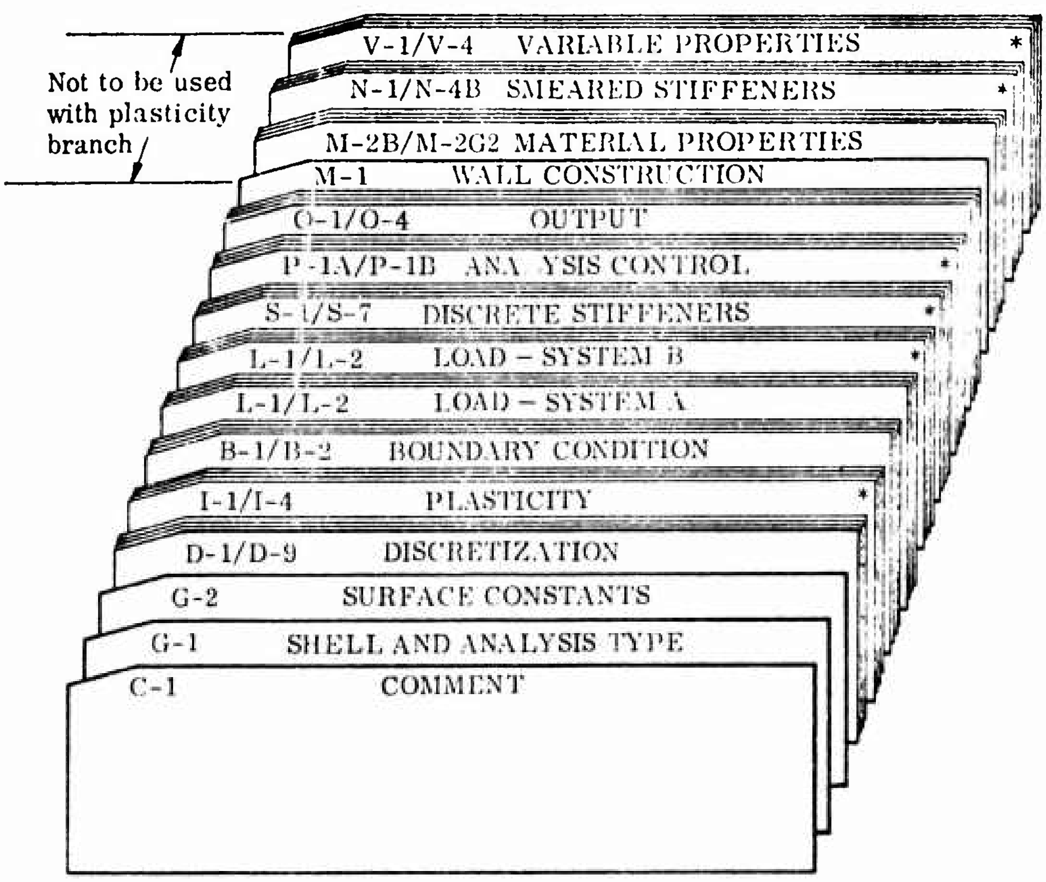
Table 7 (Cont.)

```
C                    SMEARED STIFFENERS
      COMMON /SHEAR/ E1,U1,OI1,D1,AK1,T1,H1,A1,SI1,S1,EX11,
     1                E2,U2,OI2,D2,AK2,T2,H2,A2,SI2,S2,EX22
C
C
C        WHEN IWALL=1,  SMEARED STIFFENERS MAY NOT BE SPECIFIED
C     BY INPUT CARDS (NSTRI AND NRING MUST BE ZERO ON M-1 CARD).
C     HOWEVER,  SMEARED STIFFENERS MAY BE SPECIFIED AFTER USE OF
C     EITHER OPTION A OR OPTION B IN SUBROUTINE WALL.  SET
C     VARIABLES IN COMMON BLOCK /SHEAR/ (SEE N-1 TO N-4R CARDS
C     IN SECTION 6),  THEN CALL STIFF (STIFF ADDS STIFFNESS OF
C     SMEARED STIFFENERS TO CCC MATRIX).
C
C
C     EXAMPLE UTILIZING OPTION A.
C
      PI = 3.14159
      IF (X .LT. PI / 4.0)  GO TO 10
      CT7 = 0.05
      E7 = 10000000.0
      U7 = 0.3
      CC7 = 0.4
      CH7 = 0.5
      CO7 = 0.4
      CB7 = 2.0
      Z = 0.275
      TS = 0.05
      ES = 10000000.0
      US = 0.3
      PHI = 0.5
*     ANC = 1.0
      CALL CFB(7)
      GO TO 20
   10 AT = 0.1
      EX1 = 10000000.0
      XN1 = 0.3
      Z = 0.05
      EY1 = 10000000.0
      G = EX1 / (2.0 * (1.0 + XN1))
      E1 = 10000000.0
      U1 = 0.3
      OI1 = 0.0
      F = PI / 4.0 - X / 10.0
      R1 = 10.0 * COS(F)
      D1 = PI * R1 / 20.0
      AK1 = 1.0
      T1 = 0.2
      H1 = 2.4 * X / PI
      CALL CFB(2)
      CALL STIFF
   20 CONTINUE
      RETURN
      END
```

Section 6

INPUT DESCRIPTION

There are two main branches of the program, one with and one without plasticity. The plasticity branch cannot be used with variable material properties and is restricted to isotropic wall construction. Figure 6-1 shows the data deck format. Table 8 is a minimanual summarizing the input cards. Figure 6-2 shows the sign convention for stress and moment resultants.



*Not always required

Fig. 6-1 STAGS Program – Data Deck Format

Table 8

STAGS PROGRAM MINIMANUAL

INPUT DESCRIPTION

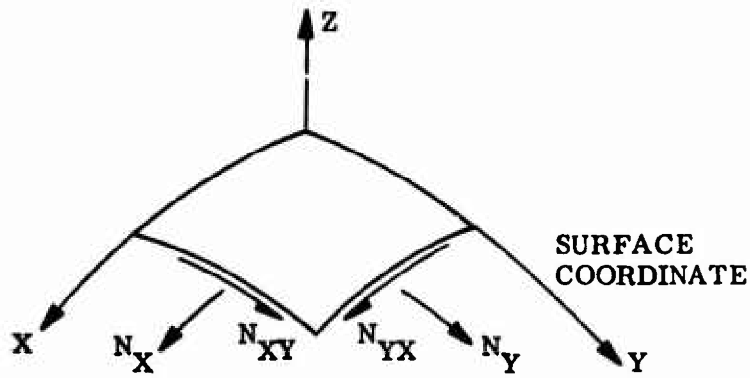
Item	Symbol	Format
C-1	COMENT (I), I=1,12	12A6
G-1	NSHELL,INDIC,NLOAD,NCHK	4I5
G-2	PROP(I), I=1,8	8E10.6
D-1	NR,NC,NRW1,NRW2,NCL1,NSTFF	6I5
	(INCLUDE ITEMS D-2 TO D-4 ONLY IF NR.EQ.0)	
D-2	NNX	I5
D-3	SEGLX(I), I=1,NNX	8E10.6
D-4	NSEGX(I), I=1,NNX	16I5
	(INCLUDE ITEMS D-5 TO D-7 ONLY IF NC.EQ.0)	
D-5	NNY	I5
D-6	SEGLY(J), J=1,NNY	8E10.6
D-7	NSEGY(J), J=1,NNY	16I5
	(INCLUDE ITEM D-8 ONLY IF NR.LT.0)	
D-8	X(I), I=1,NR	8E10.6
	(INCLUDE ITEM D-9 ONLY IF NC.LT.0)	
D-9	Y(J),J=1,NC	8E10.6
	(INCLUDE ITEMS I-1 TO I-4 ONLY IF INDIC.EQ.3)	
I-1	AE,XNU,AT,AK2	4 E10.6
I-2	NL,IC	2I5
I-3	S(I), I=1,IC	8E10.6
I-4	E(I), I=1,IC	8E10.6
B-1	IBLN(I), I=1,4	4I5
	(INCLUDE ITEM B-2 ONLY IF IBLN(I).EQ.0)	
B-2	ICOND(I), I=1,4	4I5
L-1	NN(K),LFLG(K),STLD(K),LSTP(K),MXL(K)	2I5,3E10.6
L-2	PZ,PY,PX,JZ,JY,JX,L,M	3E10.6,5I5
	(REPEAT ITEM L-2 NN TIMES)	
	(REPEAT ITEMS L-1 AND L-2 FOR K=1,2)	
	(IF NO DISCRETE STIFFENERS ARE PRESENT, GO TO P-1A CARD FOR BIFURCATION, P-1B CARD FOR NONLINEAR, AND O-1 CARD FOR LINEAR STRESS ANALYSIS)	
S-1	IRGS,ITRN,IRSO,ISTR,ITSN,ISSO	6I5
	(INCLUDE ITEMS S-2 TO S-4 ONLY IF IRGS.GT.0)	
S-2	IRN(I),IRTP(I),IRNA(I),IRNB(I),XRN(I),Y1RN(I),Y2RN(I)	4I5,3E10.6
	(REPEAT ITEM S-2 FOR I=1,IRGS)	
S-3	ERN(J),ZARN(J),ZIXRN(J),ZIZRN(J),ZJRN(J),EZRN(J),ZK1	7E10.6
	(REPEAT ITEM S-3 FOR J=1,ITRN)	
	(INCLUDE ITEM S-4 ONLY IF IRSO.GT.0)	
S-4	Z1,X1,Z2,X2,Z3,X3,Z4,X4	8E10.6
	(REPEAT ITEM S-4 FOR J=1,ITRN)	
	(INCLUDE ITEMS S-5 TO S-7 ONLY IF ISTR.GT.0)	

Table 8 (Cont.)

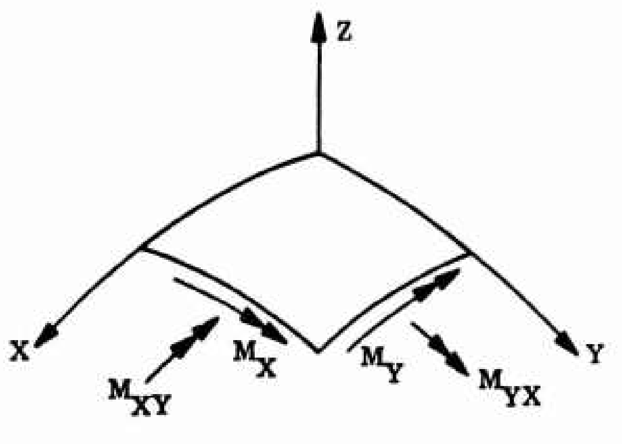
Item	Symbol	Format
S-5	ISN(I), ISTP(I), ISTA(I), ISTB(I), YSN(I), X1SN(I), X2SN(I)	4I5,3E10.6
	(REPEAT ITEM S-5 FOR I=1,ISTR)	
S-6	ESN(J), XASN(J), XIYSN(J), XIZSN(J), XJSN(J), EZSN(J), XK1	7E10.6
	(REPEAT ITEM S-6 FOR J=1,ITSN)	
	(INCLUDE ITEM S-7 ONLY IF ISSO.GT.0)	
S-7	Z1,Y1,Z2,Y2,Z3,Y3,Z4,Y4	8E10.6
	(REPEAT ITEM S-7 FOR J=1,ITSN)	
	(INCLUDE ITEMS P-1A TO P-1A2 ONLY FOR BIFURCATION ANALYSIS)	
P-1A	DELBIF, SHIFT, IBOND, ISHIFT, ITERAT	2E10.6,3I5
	(INCLUDE ITEM P-1A1 ONLY IF IBOND.EQ.1)	
P-1A1	JBLN(I), I=1,4	4I5
P-1A2	JCOND(I), I=1,4	4I5
	(INCLUDE ITEM P-1B ONLY FOR NONLINEAR ANALYSIS)	
P-1B	DELX, WUND, ISTART, ISEC, ICUT, INEWT, ISTRAT	2E10.6,5I5
O-1	IPR1, IPR2, IPR3, IPX, IPY, IPRD, IPRS, IPLOT, IHIST	9I5
	(INCLUDE ITEM O-2 ONLY IF IPX.GT.0)	
O-2	XPL(I), I=1,IPX	8E10.6
	(INCLUDE ITEM O-3 ONLY IF IPY.GT.0)	
O-3	YPL(I), I=1, IPY	8E10.6
	(INCLUDE ITEM O-4 ONLY IF IHIST. GT.0)	
O-4	X1, Y1, X2, Y2, X3, Y3, X4, Y4	8E10.6
	(IF INDIC.EQ.3 NO MORE CARDS ARE REQUIRED)	
M-1	IWALL, NSTRI, NRING, IP, IM, JM	6I5
	(INCLUDE ITEM M-2B ONLY IF IWALL.EQ.2)	
M-2B	AT, EX1, XNU, Z, EY1, G	6E10.6
	(INCLUDE ITEMS M-2C1 AND M-2C2 ONLY IF IWALL.EQ.3)	
M-2C1	T3, E3, U3, TH, A, B, H3, AK3	8E10.6
M-2C2	Z	E10.6
	(INCLUDE ITEMS M-2D1 AND M-2D2 ONLY IF IWALL.EQ.4)	
M-2D1	EF, EM, UF, UM, Z, LAYERS	5E10.6,I5
M-2D2	TT(J), XX(J), BE(J), O(J)	4E10.6
	(REPEAT ITEM M-2D2 FOR J=1, LAYERS)	
	(INCLUDE ITEMS M-2E1 AND M-2E2 ONLY IF IWALL.EQ.5)	
M-2E1	Z, LAYS	E10.6,I5
M-2E2	TL(J), EX5(J), EY5(J), UXY(J), G5(J)	5E10.6
	(REPEAT ITEM M-2E2 FOR J=1, LAYS)	
	(INCLUDE ITEM M-2F ONLY IF IWALL.EQ.6)	
M-2F	CT6, E6, U6, CC6, CH6, CD6, CB6, Z	8E10.6
	(INCLUDE ITEMS M-2G1 AND M-2G2 ONLY IF IWALL.EQ.7)	
M-2G1	CT7, E7, U7, CC7, CH7, CD7, CB7, Z	8E10.6

Table 8 (Cont.)

Item	Symbol	Format
M-2G2	TS, ES, US, PHI, ANC	5E10.6
	(INCLUDE ITEMS N-1 TO N-2B ONLY IF NSTRI.EQ.1)	
N-1	E1, U1, OI1, D1, AK1	5E10.6
	(INCLUDE ITEM N-2A ONLY IF OI1.EQ.0)	
N-2A	T1, H1	2E10.6
	(INCLUDE ITEM N-2B ONLY IF OI1.EQ.1)	
N-2B	A1, SI1, XI1, S1, EZ1, H1	6E10.6
	(INCLUDE ITEMS N-3 TO N-4B ONLY IF NRING.EQ.1)	
N-3	E2, U2, OI2, D2, AK2	5E10.6
	(INCLUDE ITEM N-4A ONLY IF OI2.EQ.0)	
N-4A	T2, H2	2E10.6
	(INCLUDE ITEM N-4B ONLY IF OI2.EQ.1)	
N-4B	A2, SI2, XI2, S2, EZ2, H2	6E10.6
	(INCLUDE ITEMS V-1 TO V-4 ONLY IF IWALL.EQ.9)	
V-1	TD, Z	2E10.6
V-2	XM(I), I=1, IM	8E10.6
V-3	YM(J), J=1, JM	8E10.6
V-4	((TDEG(L, M, N), EX(L, M, N), EY(L, M, N), U(L, M, N), G(L, M, N,), A1(L, M, N), A2(L, M, N), L=1, IP), M=1, MSTA)	7E10.6
	WHERE MSTA=IM IF NC.GT.NR, ELSE MSTA=JM.	
	(REPEAT ITEM V-4 FOR N=1, NSTA, WHERE NSTA=JM IF NC.GT.NR, ELSE NSTA=IM)	



STRESS RESULTANTS



MOMENT RESULTANTS

Fig. 6-2 Sign Convention for Stress and Moment Resultants

C-1 Comment Card

The Comment or Case Title card may contain any Hollerith Text. This comment is printed at the beginning of the output for the case.

Variable	Format	Columns	Description
COMENT	12A6	1–72	Case Title

GEOMETRY

G-1 Shell and Analysis Type Definition Card

The card is used to define the surface type of the shell by a single intege: , ıtry. It also serves to indicate the type of analysis desired.

Variable	Format	Columns	Description
NSHELL	I5	1–5	Shell type may vary from 1 through N as described under G-2 card. NSHELL = 1, Cylinder 2, Cone/Annular Plate 3, Plate 4, Sphere 5, Paraboloid 6, Elliptic Cylinder 7, Ellipsoid 8, Torus 9, Hyperboloid 10, Elliptic Cone 11, 12, User written subroutine
INDIC	I5	6–10	0 – Linear solution only 1 – Bifurcation analysis 2 – Nonlinear analysis (elastic) 3 – Nonlinear analysis (plastic)
NLOAD	I5	11–15	Number of load systems to be applied independently (presently NLOAD is restricted to 1 or 2) (Section 3.7, Loading). Note: Load cards for system B omitted if NLOAD = 1.
NCHK	I5	16-20	0 – Execute (no 3D plot) 1 – Do not execute. Provide input data check and 3D plot 2 – Execute and provide 3D plot

G-2 Surface Constants Card

This card will contain the various measurements related to the particular type of surface defined by the NSHELL integer on the G-1 card.

Variable	Format	Columns	Description
PROP (I)	8E 10.6	1–80	Surface properties depending on NSHELL (Fig. 6-3). All angles in degrees for NSHELL less than 11. See Table 4, p. 5-7, for definition of PROP (I) when NSHELL greater than 10.

Note: X-ξ and Y-η coordinates designation are synonymous for all shell types.

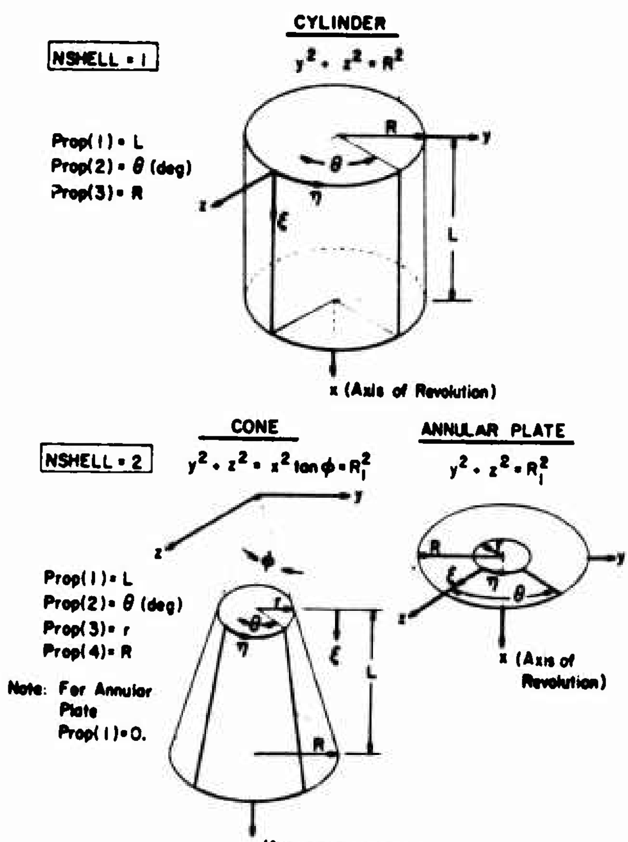
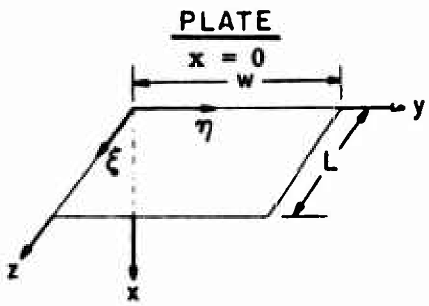


Fig. 6-3 Types of Shell Surfaces Defined by NSHELL Integer

NSHELL = 3

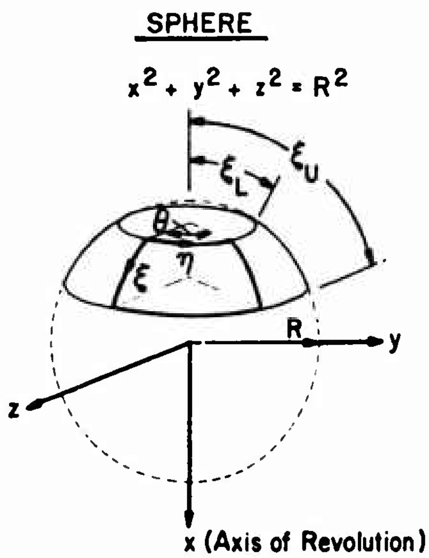
Prop(1) = L
Prop(2) = w



Note: x-y and X-Y are not synonymous. (See Nomenclature.)

NSHELL = 4

Prop(1) = ξ_L (deg)
Prop(2) = ξ_U (deg)
Prop(3) = θ (deg)
Prop(4) = R



NSHELL = 5

Prop(1) = L
Prop(2) = θ (deg)
Prop(3) = R
Prop(4) = R_1

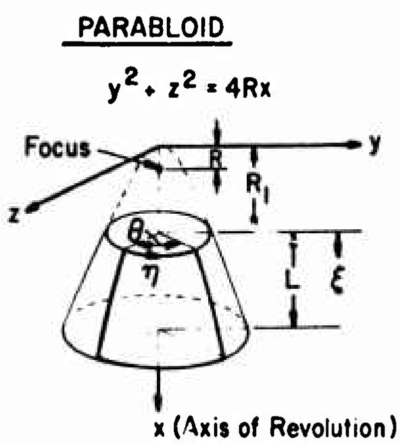


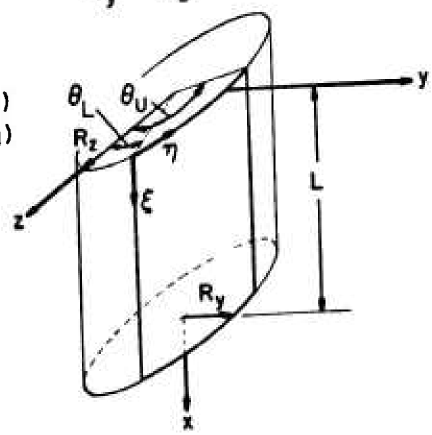
Fig. 6-3 (Cont.)

ELLIPTIC CYLINDER

NSHELL = 6

$$\frac{y^2}{R_y^2} + \frac{z^2}{R_z^2} = 1$$

Prop(1) = L
Prop(2) = θ_L (deg)
Prop(3) = θ_U (deg)
Prop(4) = R_z
Prop(5) = R_y

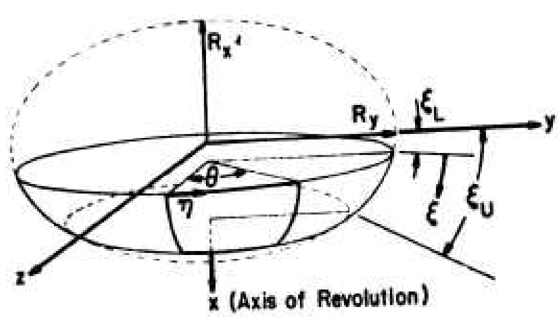


ELLIPSOID

NSHELL = 7

$$\frac{x^2}{R_x^2} + \frac{y^2}{R_y^2} + \frac{z^2}{R_y^2} = 1$$

Prop(1) = ξ_L (deg)
Prop(2) = ξ_U (deg)
Prop(3) = θ (deg)
Prop(4) = R_y
Prop(5) = R_x



TORUS

NSHELL = 8

Prop(1) = ξ_L (deg)
Prop(2) = ξ_U (deg)
Prop(3) = θ (deg)
Prop(4) = R
Prop(5) = R_1

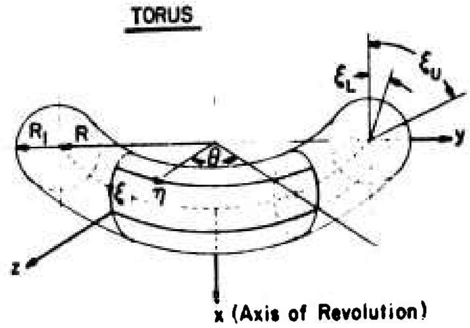


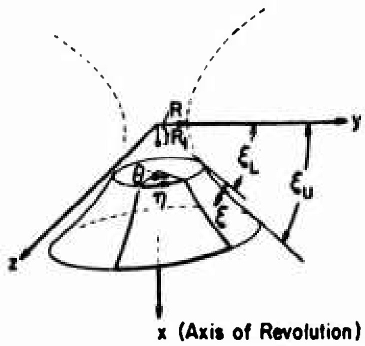
Fig. 6-3 (Cont.)

HYPERBOLOID

$$-\frac{x^2}{R_l^2}+\frac{y^2}{R^2}+\frac{z^2}{R^2}=1$$

$\boxed{\text{NSHELL} = 9}$

Prop(1) = ξ_L (deg)
Prop(2) = ξ_U (deg)
Prop(3) = θ (deg)
Prop(4) = R
Prop(5) = R_l



ELLIPTIC CONE

$$\frac{y^2}{R_y^2}+\frac{z^2}{R_z^2}=x^2$$

$\boxed{\text{NSHELL} = 10}$

Prop(1) = L
Prop(2) = L_e
Prop(3) = θ_L (deg)
Prop(4) = θ_U (deg)
Prop(5) = R_z
Prop(6) = R_y

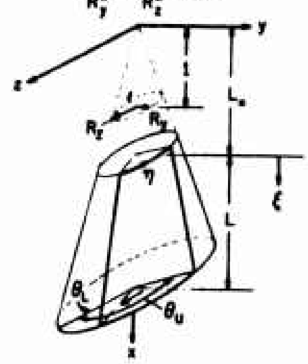


Fig. 6-3 (Cont.)

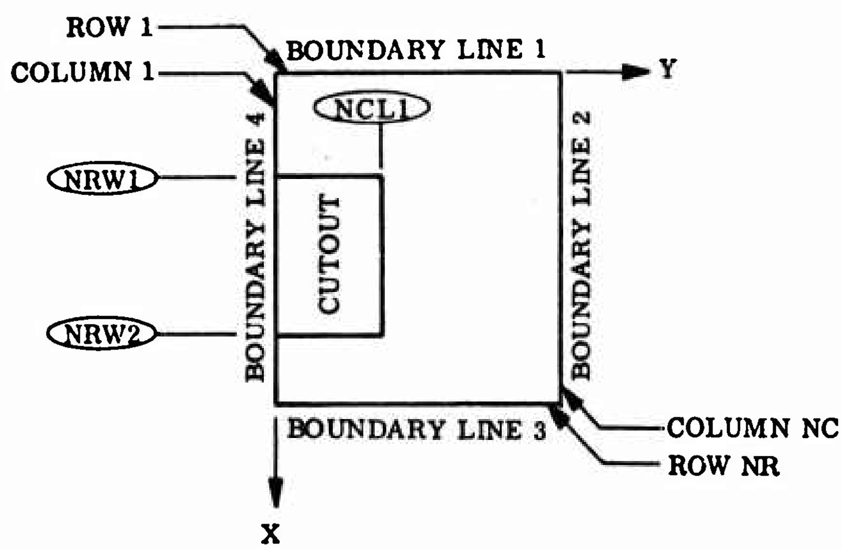
NSHELL = 11

User supplied subroutine (ORTH) for shells described by orthogonal surface coordinate lines and not included in NSHELL = 1 through 10 (see Section 5.4).

NSHELL = 12

User supplied subroutine (UNORTH) for shells described by nonorthogonal surface coordinate lines and not included in NSHELL = 1 through 10 (see Section 5.5).

DISCRETIZATION



D-1 Mesh Definition Card

Variable	Format	Columns	Description
NR	I5	1–5	Number of rows (Stations along the X coordinate). If NR negative, spacing is variable, read in X coordinates on Card D-8. If NR = 0, spacing is constant within each of a number of segments but varies from one segment to another (see D-2 card).
NC	I5	6–10	Number of columns (Stations along the Y coordinate). If NC negative, spacing is variable, read in Y coordinates on Card D-9. If NC = 0, spacing is constant within each of a number of segments but varies from one segment to another. (See D-5 card.)
NRW1	I5	11 – 15	Row number of one edge of cutout.
NRW2	I5	16 – 20	Row number of other edge of cutout.
NCL1	I5	21 – 25	Column number of edge of cutout.
NSTFF	I5	26 – 30	0 – No discrete stiffeners 1 – Discrete stiffeners (input data on S-1 card)

(This card continued next page.)

Note:

NRW1, NRW2, and NCL1 define a rectangular cutout adjacent to boundary line 4. In the case of no cutout, NRW1, NRW2, and NCL1 should be blank. Additional cutouts or cutouts of more general shape can be included through specification of a zero modulus of elasticity in the appropriate area (use IWALL = 1 on M-1 card).

If NR and NC are positive numbers, omit cards D-2 through D-9.

D-2 X-Segment Card

This card should be included only if NR is zero on the D-1 card.

Variable	Format	Columns	Description
NNX	I5	1-5	Number of segments in X direction with constant spacing.

D-3 X-Segment Length Definition Cards

These cards should be included only if NR is zero on the D-1 card.

Variable	Format	Columns	Description
SEGLX(I) I = 1, NNX	8E 10.6	1-80	"Length" of Segment I, (i.e., difference between extreme values of X coordinate).

D-4 X-Segment Spacing Definition Cards

These cards should be included only if NR is zero on the D-1 card.

Variable	Format	Columns	Description
NSEGX(I) I = 1, NNX	16 I5	1-80	Number of mesh spaces within Segment I.

D-5 Y-Segment Card

This card should be included only if NC is zero on the D-1 card.

Variable	Format	Columns	Description
NNY	I5	1-5	Number of segments in Y direction with constant spacing.

D-6 Y-Segment Length Definition Cards

These cards should be included only if NC is zero on the D-1 card.

Variable	Format	Columns	Description
SEGLY(J) J = 1, NNY	8E 10.6	1-80	"Length" of Segment J (i.e., difference between extreme values of Y coordinate).

D-7 Y-Segment Spacing Definition Cards

These cards should be included only if NC is zero on the D-1 card.

Variable	Format	Columns	Description
NSEGY(J) J = 1, NNY	16 I5	1-80	Number of mesh spaces within Segment J.

Unless NR or NC is negative, omit D-8 and D-9 cards.

D-8 X-Coordinate Cards

These cards should be included only if NR is negative on the D-1 card.

Variable	Format	Columns	Description
X(I) (I = 1, NR)	8E 10.6	1-80	X coordinate for Row I (Must be monotonically increasing)

D-9 Y-Coordinate Cards

These cards should be included only if NC is negative on the D-1 card.

Variable	Format	Columns	Description
Y(J) (J = 1, NC)	8E 10.6	1−80	Y coordinate for Column J (Must be monotonically increasing)

Note:

Unless the plasticity branch is used, omit cards I-1 through I-4, and go to the B-1 card.

PLASTICITY

Cards I-1 through I-4 are used only for the plasticity branch, i.e., INDIC = 3 on the G-1 card. For the elastic branch, go to the B-1 card.

I-1 Plasticity Definition Card-1

Variable	Format	Columns	Description
AE	E 10.6	1–10	Young's Modulus
XNU	E 10.6	11–20	Poisson's Ratio
AT	E 10.6	21–30	Shell Thickness
AK2	E 10.6	31–40	Square of the ellipse-ratio of yield surface. Usually AK2 = 3.0

If the elastic branch is used, corresponding information is read at a different place.

I-2 Plasticity Definition Card-2

Variable	Format	Columns	
NL	I5	1–5	Number of points across wall thickness. Must be odd number and not less than 3 or more than 9.
IC	I5	6–10	Number of material components (Number of points defined on the stress-strain curve). (Fig. 6-4) IC ≤ 10

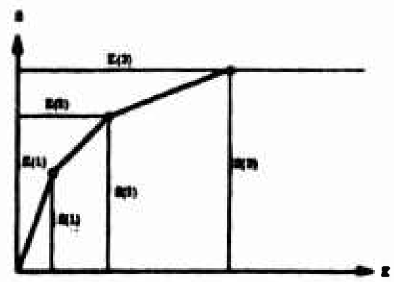


Fig. 6-4 Stress-Strain Curve

I-3 Plasticity Definition Card-3

Variable	Format	Columns	Description
S(I) I = 1, IC	E 10.6	1-80	Stress values on the stress-strain curve. See Fig. 6-4.

I-4 Plasticity Definition Card-4

Variable	Format	Columns	Description
E(I) I = 1, IC	E 10.6	1-80	Corresponding strain values on the stress-strain curve. See Fig. 6-4.

Note: E(1) need not be punched. It will be computed by the program as

$$E(1) = S(1)/AE$$

BOUNDARY CONDITION

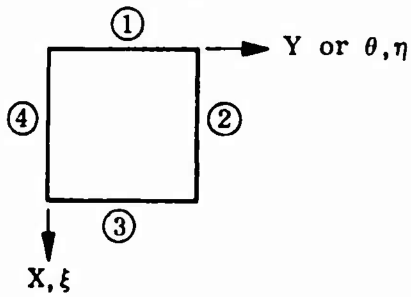
B-1 Boundary Condition Card-1

Do not specify loads and displacements which are in conflict, such as simple support and specified tangential displacement.

For bifurcation buckling (INDIC = 1), the conditions defined here will apply to the pre-buckling displacements. If the boundary conditions for incremental displacement are different, they will be specified on the P-1A, P-1A1, and P-1A2 cards below.

Boundary lines are numbered 1 through 4 according to the following convention:

Row 1 – Boundary line 1
Column NC – Boundary line 2
Row NR – Boundary line 3
Column 1 – Boundary line 4



Note: Side 1 X = 0, Side 4 Y = 0.

Variable	Format	Columns	Description
IBLN(I) I = 1,4	4 I5	1–20	Boundary code for Line I. 0 – Specified on B-2 cards 1 – Simple support (defined below) 2 – Clamped (defined below) 3 – Unrestrained 4 – Symmetry (defined below) 5 – Anti-symmetry (defined below) 6 -- Closed shell in the η direction (see comment below and pages 2-2 and 2-3).

Here simple support means $w = v = 0$ on lines 1 and 3
$w = u = 0$ on lines 2 and 4

(This card continued next page.)

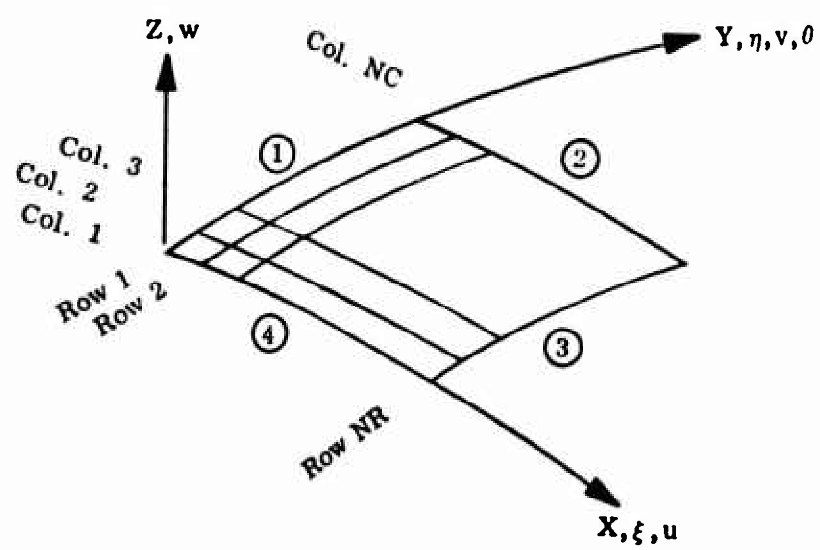
clamped means that all displacements and rotations are restrained and cannot be used for loaded edge

Symmetry implies	$u = \beta = 0$ on lines 1 and 3
	$v = \beta = 0$ on lines 2 and 4
Antisymmetry implies	$v = w = u,_x = 0$ on lines 1 and 3
	$u = w = v,_y = 0$ on lines 2 and 4

The option IBLN = 6 indicates a closed shell. In that case, set IBLN(2) = IBLN(4) = 6. Do not set IBLN (1) and IBLN(3) equal to six. Boundary lines of zero length (e.g., apex) have not been incorporated in the program.

B-2 Boundary Condition Cards. Use only if IBLN(I) = 0 on the B-1 card. One B-2 card for each I with IBLN = 0. The cards refer to boundary lines with increasing numbers.

Variable	Format	Columns	Description
ICOND(J) J = 1, 4	4 I5	1 – 20	Freedom of movement of Line I in regard to the w, v, u, and β displacements, where w, v, and u are displacements in the direction of the coordinates Z, Y, and X, respectively, and β is the rotation around a tangent to the edge 1 – Free to move 0 – No movement



LOADS

A Load System is defined by an L-1 card followed by the appropriate L-2 cards. If NLOAD on the G-1 card is 2, two Load Systems are required – one for Load System A and another for Load System B. The input data cards for Load System A must be completed before the input data cards for Load System B begin. The use of Load System B in bifurcation analysis is explained in Section 3.7, Loading.

A base load for each Load System is defined by the set of L-2 cards together with any loads that may be introduced by a user-wrtten subroutine (USRLD). STLD, LSTP, and MXL refer to the base load of the system.

L-1 Card

Variable	Format	Columns	Description
NN	I5	1 – 5	Number of L-2 cards required to describe the Load System.
LFLG	I5	6 – 10	User-Load Flag. 0 – User does not have own subroutine to define Load System. 1 – User has own subroutine to define Load System.
STLD	E 10.6	11 – 20	Starting Load Factor. (The initial load is STLD times the base load.) For nonlinear analysis, STLD is the current starting Load Factor. (See sample case 1 Second Run, Page 7-10). For linear analysis, the total load equals the initial load. For bifurcation analysis, the critical load is the eignevalue times the initial load.
LSTP	E 10.6	21 – 30	Load Step Increment. (The load increment is LSTP times the base load.) Meaningful only for nonlinear analysis.
MXL	E 10.6	31 – 40	Maximum Load. (The maximum load is MXL times the base load.) Meaningful only for nonlinear analysis. May be used to freeze either Load System A or Load System B.

L-2 Cards

Do not prescribe loads and displacements on the same L-2 card.

Variable	Format	Columns	Description
PZ, PY, PX	3 E 10.6	1 – 30	Load Element. (Positive outward and toward increasing X and Y.)
JZ, JY, JX	3 I5	31 – 45	Refer to PZ, PY, and PX, respectively -1 – Displacement 0 – Omit 1 – Point Force 2 – Line Load along Row 3 – Line Load along Column 4 – Pressure Load 5 – Live Pressure Load (Use only for L = M = 0.)
L, M	2 I5	46 – 55	Row and Column number of the mesh-point where the particular load element is applied.

Note:

When the row number is entered as zero, the load element is assumed to act at every mesh-point on the column indicated by the column number. If the column number is entered as zero, the load élement is assumed to act at every mesh-point on the row indicated by the row number. If both row and column numbers are zero, then the load element is assumed to act at each mesh-point.

For inplane displacements, only uniform displacement of one of the four boundary lines may be prescribed. That is, either L or M must be zero.

The energy method on which the program is based assumes that loads and structure comprise a conservative system. Because there is some controversy about the requirements for a system with variable load to be conservative, the live load option is internally suppressed unless the pressure is uniform.

Here follows L-1 and L-2 cards for Load System B if NLOAD (G-1 card) is equal to 2.

DISCRETE STIFFENERS

S-1 Discrete Stiffener Definition Card (Ring is defined as a stiffener on constant X-coordinate; stringer is defined as a stiffener on constant Y-coordinate.) Note that "smeared" stiffener can be defined in lieu of or in addition to the discrete stiffeners (N-1 through N-4B cards below).

Note:

If no discrete stiffeners are present, NSTFF equal zero on D-1 card; go to the P-1A card for bifurcation analysis, to P-1B card for nonlinear analysis, and to the O-1 card for linear stress analysis.

Variable	Format	Columns	Description
IRGS	I5	1–5	Number of rings, IRGS ≤ 80
ITRN	I5	6–10	Number of distinct type of rings. ITRN ≤ 30
IRSO	I5	11–15	Stress output will be given at IRSO points in the ring cross-section. IRSO ≤ 4
ISTR	I5	16–20	Number of stringers. ISTR ≤ 80
ITSN	I5	21–25	Number of distinct type of stringers. ITSN ≤ 30
ISSO	I5	26–30	Stress output will be given at ISSO points in the stringer cross-section. ISSO ≤ 4

If there are no rings (IRGS = 0), go to the S-5 card.

S-2 Ring Delimiter Cards

Include these cards only if IRGS on Card S-1 is not 0.

Variable	Format	Columns	Description
IRN(I)	I5	1–5	Row number of ring
IRTP(I)	I5	6–10	Ring type
IRNA(I)	I5	11–15	Starting column of ring

(This card continued next page.)

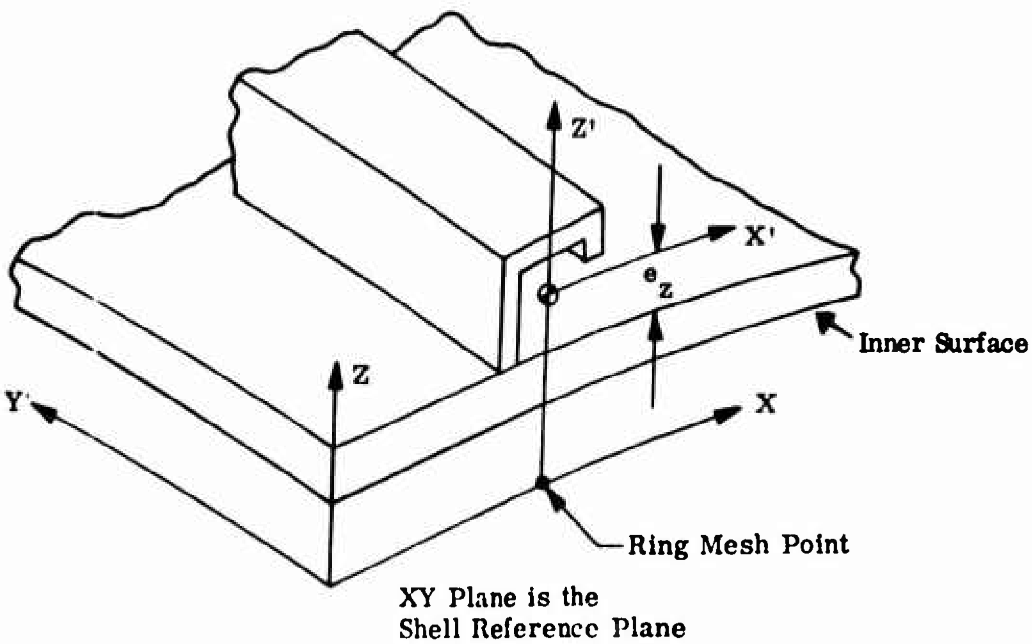
Variable	Format	Columns	Description
IRNB(I)	I5	16–20	Ending column of ring. (If IRN, IRNA, and IRNB are set equal to zero, the position of the ring is given by coordinate values rather than by row and column numbers)
XRN(I)	E 10.6	21–30	X coordinate of ring
Y1RN(I)	E 10.6	31–40	Y coordinate of start of ring
Y2RN(I)	E 10.6	41–50	Y coordinate of end of ring

Notes:

1. XRN, Y1RN, and Y2RN are optional data elements and will be used to define ring location only if IRN, IRNA, and IRNB are zero, respectively.
2. If XRN, Y1RN, and Y2RN do not coincide with a grid line, the ring will be placed (by the program) at the closest grid line.
3. Repeat this card for I = 1, IRGS.

S-3 Ring Description Cards

Include these cards only if IRGS card S-1 is not zero. Card format is 8E 10.6. One card is required for each ring type (ITRN cards).



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Variable	Format	Columns	Description
ERN(J)	E 10.6	1−10	Young's Modulus for ring type J.
ZARN(J)	E 10.6	11−20	Cross-Section Area for ring type J.
ZIXRN(J)	E 10.6	21−30	Moment of Inertia about X'-axis for ring type J.
ZIZRN(J)	E 10.6	31−40	Moment of Inertia about Z'-axis for ring type J.
ZJRN(J)	E 10.6	41−50	Torsional stiffness GJ for ring type J
EZRN(J)	E 10.6	51−60	Eccentricity in Z' direction for ring type J. Outside ring – distance from outer shell surface to ring centroid (positive in + Z direction). Inside ring – distance from inner shell surface to centroid (positive in - Z direction).
ZK1	E 10.6	61 - 70	0. – internal rings 1. – external rings

Note:
Repeat above data for J = 1, ITRN.

S-4 Ring Stress Output Card

Include these cards only if IRSO on Card S-1 is not 0.

Variable	Format	Column	Description
Z1	E 10.6	1−10	Z-coordinate for the first point with stress output.
X1	E 10.6	11−20	X'-coordinate for the same point.
Z2	E 10.6	21−30	Same information for other points with stress output. Read as many pairs of coordinates as indicated by IRSO on the S-1 card.
X2	E 10.6	31−40	
Z3	E 10.6	41−50	
X3	E 10.6	51−60	
Z4	E 10.6	61−70	
X4	E 10.6	71−80	

Note:
Repeat above data for each ring type, J = 1, ITRN.

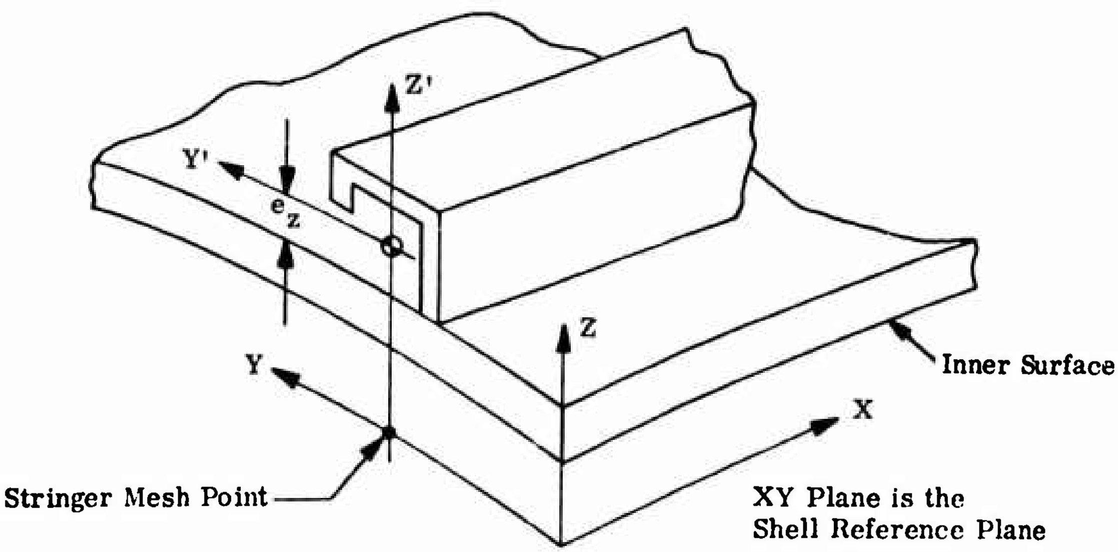
S-5 Stringer Delimiter Cards

If there are no stringers (ISTR = 0), go to the P-1A card for bifurcation analysis, P-1B for nonlinear analysis and to the O-1 card for linear stress analysis.

Include these cards only if ISTR on card S-1 is not 0.

Variable	Format	Columns	Description
ISN(I)	I5	1–5	Column number of stringer
ISTP(I)	I5	6–10	Stringer type
ISTA(I)	I5	11–15	Starting row of stringer
ISTB(I)	I5	16–20	Ending row of stringer. (If ISN, ISTA, and ISTB are set equal to zero, the position of the stringer is given by coordinate values rather than by column and row numbers).
YSN(I)	E 10.6	21–30	Y coordinate of stringer
X1SN(I)	E 10.6	31–40	X coordinate of start of stringer
X2SN(I)	E 10.6	41–50	X coordinate of end of stringer

NOTES:
1. **YSN, X1SN, and X2SN are optional data elements and will be used to define stringer location only if ISN, ISTA, and ISTB are zero, respectively.**
2. **If YSN, X1SN, and X2SN do not coincide with a grid line, the stringer will be placed (by the program) at the closest grid line.**
3. **Repeat this card for I = 1, ISTR.**



S-6 Stringer Description Cards

Include these cards only if ISTR on card S-1 is not zero. Card format is 8E10.6.

Variable	Format	Columns	Description
ESN(J)	E 10.6	1–10	Young's Modulus for stringer type J.
XASN(J)	E 10.6	11–20	Cross-section Area for stringer type J.
XIYSN(J)	E 10.6	21–30	Moment of Inertia about Y'-axis for stringer type J.
XIZSN(J)	E 10.6	31–40	Moment of Inertia about Z'-axis for stringer type J.
XJSN(J)	E 10.6	41–50	Torsional stiffness GJ for stringer type J
EZSN(J)	E 10.6	51–60	Eccentricity in Z' direction for stringer type J. Outside stringer – distance from outer shell surface to stringer centroid (positive in +Z direction). Inside stringer – distance from inner shell surface to centroid (positive in -Z direction).
XK1	E 10.6	61 – 70	0. – internal stringers 1. – external stringers

Note:

Repeat above data for J = 1, ITSN.

S-7 Stringer Stress Output Card

Include these cards only if ISSO on card S-1 is not 0.

Variable	Format	Column	Description
Z1	E 10.6	1–10	Z-coordinate for the first point with stress output.
Y1	E 10.6	11 – 20	Y'-coordinate for the same point.

(This card continued next page.)

Variable	Format	Column	Description
Z2	E 10.6	21–30	Same information for other points with stress output. Read as many pairs of coordinates as indicated by ISSO on the S-1 card.
Y2	E 10.6	31–40	
Z3	E 10.6	41–50	
Y3	E 10.6	51–60	
Z4	E 10.6	61–70	
Y4	E 10.6	71–80	

Note:

Repeat above data for each stringer type, J = 1, ITSN.

For nonlinear analysis, go to the P-1B card; for linear stress analysis, go to the O-1 card.

ANALYSIS CONTROL

P-1A Parameter Card: Used for bifurcation analysis, INDIC = 1 on G-1 card.

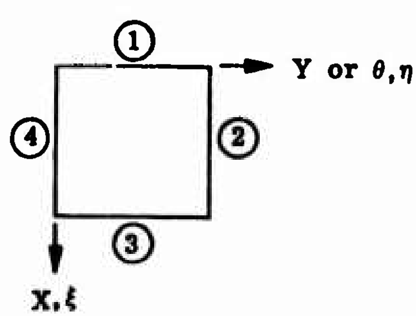
Variable	Format	Columns	Description
DELBIF	E 10.6	1–10	Error tolerance in power-iteration for eigenvalue. If DELBIF is zero or blank, the error tolerance .0001 is used.
SHIFT	E 10.6	11–20	Initial eigenvalue shift, if any.
IBOND	I5	21–25	1 – The boundary conditions for incremental displacements are different from the prebuckling displacements. 0 – The boundary conditions are the same for incremental and prebuckling displacements.
ISHIFT	I5	26–30	Number of eigenvalue shifts permitted.
ITERAT	I5	31–35	Number of inverse power iterations permitted between shifts.

If IBOND = 0, go the O-1 card.

P-1A1 Parameter Card: Incremental displacement boundary condition. Used for bifurcation analysis with different boundary conditions for incremental and prebuckling displacement (INDIC = 1 on G-1 card and IBOND = 1 on P-1A card).

Boundary lines are numbered 1 through 4 according to the following convention:

Row 1 – Boundary line 1
Column NC – Boundary line 2
Row NR – Boundary line 3
Column 1 – Boundary line 4



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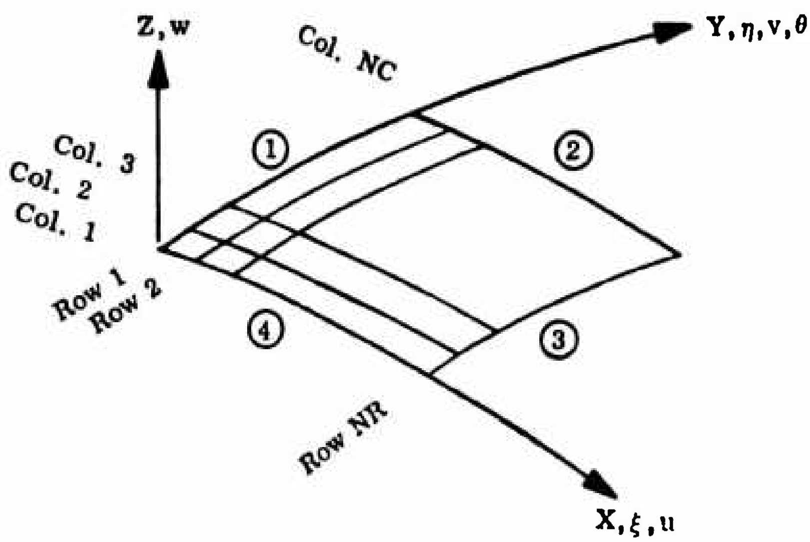
Variable	Format	Columns	Description
JBLN(I) I = 1, 4	4 I5	1–20	Boundary code for Line I. 0 – Specified on P-1A2 cards 1 – Simple support 2 – Clamped 3 – Unrestrained 4 – Symmetry 5 – Anti-symmetry 6 – Closed shell

Note: See discussion of B-1 card. JBLN(I) = 6 only if IBLN(I) = 6.

P-1A2 Parameter Cards: Used only if JBLN(I) = 0 on the P-1A1 card. One P-1A2 card for each I with JBLN = 0. The cards refer to boundary lines with increasing numbers.

Variable	Format	Columns	Description
JCOND(I) I = 1, 4	4 I5	1–20	Freedom of movement of Line I in regard to the w, v, u, and β displacements, where w, v, and u are displacements in the direction of the coordinates Z, Y, and X, respectively and β is the rotation around a tangent to the edge. 1 – Free to move 0 – No movement



Go to the O-1 card.

P-1B Parameter Card; Used for nonlinear analysis, INDIC = 2 or 3 on G-1 card.

Variable	Format	Columns	Description
DELX	E 10.6	1–10	Error tolerance.
WUND	E 10.6	11–20	Underrelaxation factor.

Note: If DELX is zero or blank, the error tolerance .0001 is used. If WUND is zero or blank, the relaxation factor of 1.0 is used initially. The relaxation factor is increased internally if convergence is monotonic but slow, and it is decreased internally if convergence is highly oscillatory. If WUND ≠ 0, the input value remains unchanged regardless of convergence.

Variable	Format	Columns	Description
ISTART	I5	21–25	Starting Code: (see also Section 7.1) 0 – Begin new case. 1 – Restart case from 1st record. 2 – Restart case from 2nd record. 3 – Restart from 3rd record. Last load step. Note: For plasticity (INDIC=3) ISTART is either 0 or 3. ISTART 1 or 2 cannot be used.
ISEC	I5	26–30	Number of CPU seconds of run time at which run should be terminated and data saved on restart tape. (See Strategy, Section 4)
ICUT	I5	31–35	Total number of times step size may be cut. (See Strategy, Section 4).
INEWT	I5	36–40	Number of Newton steps which may be taken. (See Strategy, Section 4)
ISTRAT	I5	41–45	Number of times step size is cut between Newton steps. (See Strategy, Section 4)

OUTPUT

O-1 Output Definition Card

Variable	Format	Columns	Description
IPR1	I5	1–5	Print displacement solutions every IPR1 load step.*
IPR2	I5	6–10	Print stress resultants every IPR2 load step.*
IPR3	I5	11–15	Print stresses every IPR3 load step.*

*Note: With IPR1, IPR2, or IPR3 equal zero corresponding output is suppressed. This option can be used also in the linear or bifurcation buckling case if one wants to suppress all regular output in favor of the selected output defined by IPX, IPY.

Variable	Format	Columns	Description
IPX	I5	16–20	Number of selected rows along which displacements and/or stress resultant solutions will be printed. IPX ≤ 50
IPY	I5	21–25	Number of selected columns along which displacements and/or stress resultant solutions will be printed. IPY ≤ 50
IPRD	I5	26–30	Print selected displacement solutions every IPRD load step.
IPRS	I5	31–35	Print selected stress resultant solutions every IPRS load step.
IPLOT	I5	36-40	See Section 8 for details. 0 – No plots. 1 – Plot and print special output (see notes below). Plot selected output (if any). 2 – Plot special output. Plot selected output (if any).
IHIST	I5	41-45	See Section 8 for details. The number of selected points for which history plots are required. If no history plots are required, set IHIST=0. IHIST ≤4.

(This card continued next page.)

Notes: 1. Plot means data will be saved on special tape for use with postprocessor.
2. Special output;
If INDIC=0 (on the G-1 card) – All displacements and stress resultants.
If INDIC=1 – All prebuckling displacements and stress resultants and all displacements in the buckling mode.
If INDIC=2 or 3 – All displacements and stress resultants for the last load-step and the difference between the last two displacement solutions (collapse mode).
3. For IPLOT=1 or 2 and INDIC=2 or 3, a history plot for the maximum displacement is provided in addition to IHIST plots.
4. An effort was made to eliminate duplication of output printing.

O-2 Selected Output Along Rows

Variable	Format	Columns	Description
XPL(I) I = 1, IPX	8E 10.6	1–80	X coordinates of rows along which selected output is desired.

Note: Include these cards only if IPX on card O-1 greater than zero (selected output is requested). Card format is 8E 10.6, therefore more than one O-2 card might be necessary to input all X coordinates.

O-3 Selected Output Along Columns

Variable	Format	Columns	Description
YPL(I) I = 1, IPY	8E 10.6	1–80	Y coordinates of columns along which selected output is desired.

Note: Include these cards only if IPY on card O-1 greater than zero (selected output is requested). Card format is 8E 10.6, therefore more than one O-3 card might be necessary to input all Y coordinates.

O-4 Selected Points for History Plot

This card should be included only if IHIST on card O-1 is greater than zero.

The number of entries on this card will be equal to 2 × IHIST.

Variable	Format	Columns	Description
X1	E10.6	1–10	X – Coordinate for first mesh point at which history data are saved.
Y1	E10.6	11–20	Y – Coordinate for first mesh point at which history data are saved.
X2	E10.6	21–30	
Y2	E10.6	31–40	
X3	E10.6	41–50	
Y3	E10.6	51–60	
X4	E10.6	61–70	
Y4	E10.6	71–80	

If the plasticity branch is used there are no additional cards to be read.

WALL CONSTRUCTION

M-1 Wall Type Card

Cards on which data are to be read for the different types of shell wall are given in parentheses.

Variable	Format	Columns	Description
IWALL	I5	1–5	1–A user-written subroutine, WALL, is going to be provided. (Except for the options under IWALL = 8 or 9, this is the only way to introduce variable properties). Notice that the subroutines corresponding to IWALL = 2 through 7 may be called from WALL. Input is complete. 2–Monocoque shell (may be orthotropic). (M-2B) 3–Skew-stiffened shell. (M-2C1, M-2C2) 4–Layered, fiber-wound shell. (M-2D1, M-2D2) 5–Layered shells, layers may be orthotropic. (M-2E1, M-2E2) 6–Corrugated shell. (M-2F) 7–Corrugated shell with smooth skin. (M-2G1, M-2G2) 8–Orthotropic shell. (Temperature and wall properties are defined by user written MATER subroutine and may vary through the thickness as well as with shell coordinates.) No more cards to read unless NSTRI or NRING below equals 1. 9–Orthotropic shell. (Temperature and wall properties are read at selected mesh points and may vary through the thickness as well as with shell coordinates.)(V-1, V-2, V-3, V-4)

(This card continued next page.)

Variable	Format	Columns	Description
NSTRI	I5	6−10	0 – No smeared stringers. 1 – Smeared stringers (input data on card N-1).
NRING	I5	11−15	0 – No smeared rings 1 – Smeared rings (input data on card N-3).

Unless IWALL is 8 or 9, this card is complete.

Discrete stiffeners can be added as desired above (S-1 through S-7).

Variable	Format	Columns	Description
IP	I5	16−20	Number of points across wall thickness. Must be odd number and not less than 3 or more than 9. Omit unless IWALL = 8 or 9.
IM	I5	21−25	Number of rows selected for input of temperature or properties. Omit unless IWALL = 9. IM $\geq$ 2.
JM	I5	26−30	Number of columns selected for input of temperature or properties. Omit unless IWALL = 9. JM $\geq$ 2.

MATERIAL PROPERTIES

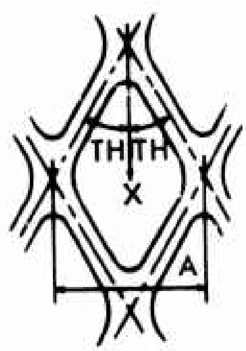
M-2B (IWALL = 2, Monocoque Shell)

Note: If shell is isotropic, punch only the first four fields on this card. For an orthotropic shell with EY1 = 0, punch a small but nonzero value for EY1.

Variable	Format	Columns	Description
AT	E 10.6	1–10	Wall thickness.
EX1	E 10.6	11–20	Young's Modulus in X-direction.
XNU	E 10.6	21–30	Poisson's Ratio – μ_{xy} , $(\mu_{xy} E_x = \mu_{yx} E_y)$
Z	E 10.6	31–40	Distance from reference surface to shell midsurface (positive if the midsurface is outside of the reference surface, i.e., Z coordinate for midsurface is positive).
EY1	E 10.6	41–50	Young's Modulus in Y-direction (however shell is assumed isotropic if EY1 = 0).
G	E 10.6	51–60	Shear Modulus.

M-2C1 (IWALL = 3, skew-stiffened shell)

Variable	Format	Columns	Description
T3	E 10.6	1–10	Wall thickness
E3	E 10.6	11–20	Young's modulus
U3	E 10.6	21–30	Poisson's ratio
TH	E 10.6	31–40	Angle (deg) between stiffeners and X-coordinate lines
A	E 10.6	41–50	Stiffener spacing (along Y-coordinate lines)
B	E 10.6	51–60	Stiffener thickness
H3	E 10.6	61–70	Stiffener height
AK3	E 10.6	71–80	0.– inside stiffening 1.– outside stiffening



M-2C2 (IWALL = 3, skew-stiffened shell)

Variable	Format	Columns	Description
Z	E 10.6	1–10	Distance from reference surface to skin midsurface (positive if the midsurface is outside the reference surface, i.e., at positive Z-coordinate).

M-2D1 (IWALL = 4, layered, fiber-wound shell)

Variable	Format	Columns	Description
EF	E 10.6	1–10	Young's modulus of fibers
EM	E 10.6	11–20	Young's modulus of matrix
UF	E 10.6	21–30	Poisson's ratio of fibers
UM	E 10.6	31–40	Poisson's ratio of matrix
Z	E 10.6	41–50	Distance from reference surface to shell midsurface (positive if midsurface is outside the reference surface; i.e., at a positive Z coordinate)
LAYERS	I5	51–55	Number of layers. LAYERS ≤ 20.

M-2D2 (IWALL = 4, layered, fiber-wound shell)

Variable	Format	Columns	Description
TT(J)	E 10.6	1–10	Thickness of layer (inner layer has index 1, outer layer has index LAYERS)
XX(J)	E 10.6	11–20	Matrix content (by volume) in percent/100
BE(J)	E 10.6	21–30	Winding angle
Ø(J)	E 10.6	31–40	Contiguity factor

Note: Repeat Card M-2D2 for J = 1, LAYERS.

M-2E1 (IWALL = 5, layered orthotropic shells)

Variable	Format	Columns	Description
Z	E 10.6	1–10	Distance from reference surface to shell midsurface (positive if midsurface is outside of reference surface, i.e., at a positive Z coordinate).
LAYS	I5	11–15	Number of layers. LAYS ≤ 20.

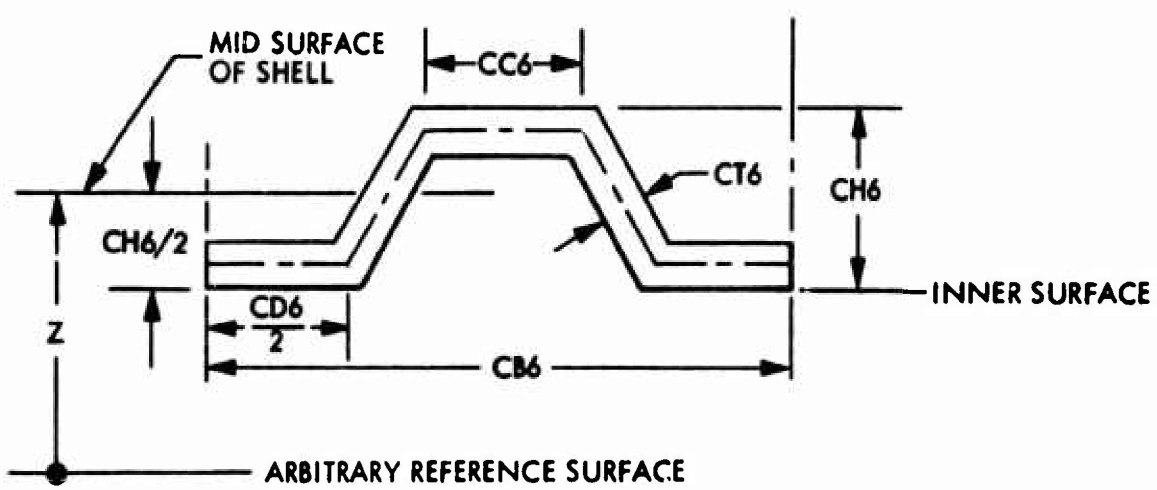
M-2E2 (IWALL = 5, layered orthotropic shell)

Variable	Format	Columns	Description
TL(J)	E 10.6	1–10	Layer thickness (inner layer No. 1, outer layer No. LAYS)
EX5(J)	E 10.6	11–20	Modulus of elasticity in x direction
EY5(J)	E 10.6	21–30	Modulus of elasticity in y direction
UXY(J)	E 10.6	31–40	Poisson's ratio – μ_{xy}, ($\mu_{xy} E_x = \mu_{yx} E_y$)
G5(J)	E 10.6	41–50	Shear modulus

Note: Repeat Card M-2E2 for J = 1, LAYS.

M-2F (IWALL = 6, Corrugated Shell)

Variable	Format	Columns	Description
CT6	E 10.6	1–10	Thickness of corrugated sheet
E6	E 10.6	11–20	Young's modulus
U6	E 10.6	21–30	Poisson's ratio
CC6	E 10.6	31–40	See figure
CH6	E 10.6	41–50	See figure
CD6	E 10.6	51–60	See figure
CB6	E 10.6	61–70	Centerline-to-centerline spacing of corrugations

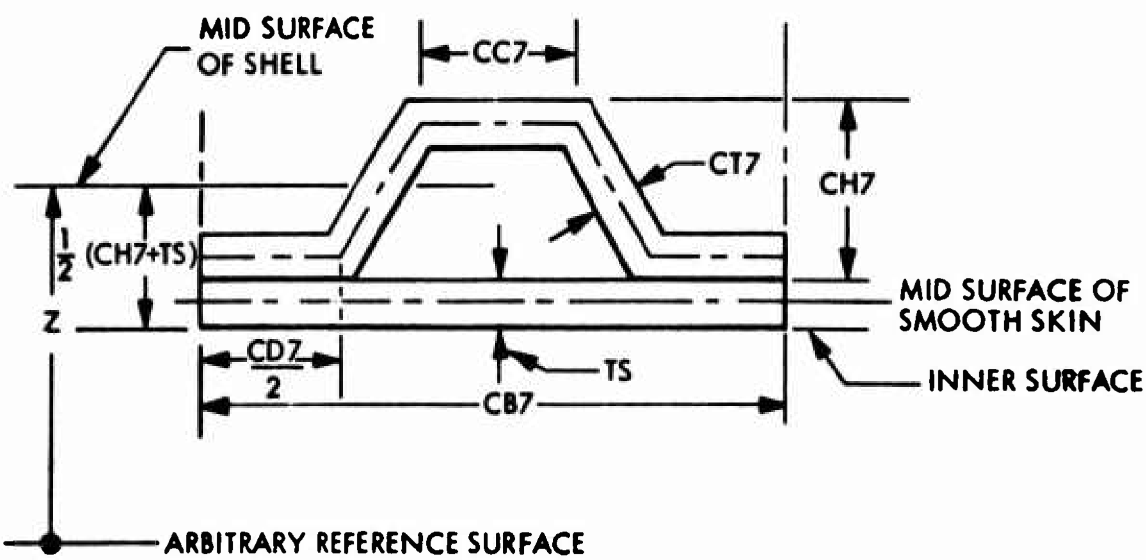


(This card continued next page.)

Variable	Format	Columns	Description
Z	E 10.6	71-80	Distance from reference surface to shell midsurface (positive if midsurface of shell is outside the reference surface, i.e., at a positive Z coordinate)

M-2G1 (IWALL = 7, Corrugated Shell with one smooth skin)

Variable	Format	Columns	Description
CT7	E 10.6	1-10	Thickness of corrugated sheet
E7	E 10.6	11-20	Young's modulus
U7	E 10.6	21-30	Poisson's ratio
CC7	E 10.6	31-40	See figure
CH7	E 10.6	41-50	See figure
CD7	E 10.6	51-60	See figure
CB7	E 10.6	61-70	Centerline-to-centerline spacing of corrugations
Z	E 10.6	71-80	Distance from reference surface to mid-surface of shell (positive if midsurface is outside of reference surface; at positive Z coordinate).



M-2G2 (IWALL = 7, Corrugated Shell with one smooth skin)

Variable	Format	Columns	Description
TS	E 10.6	1–10	Thickness of smooth skin
ES	E 10.6	11–20	Young's modulus of skin
US	E 10.6	21–30	Poisson's ratio of skin
PHI	E 10.6	31–40	Reduction factor for torsional stiffness
ANC	E 10.6	41–50	0.– Inside corrugation 1.– Outside corrugation

SMEARED STIFFENERS

Note: For smeared rectangular stiffeners, GJ is computed by the program as $H \times T^3/3$ when $H \geq T$ (somewhat unconservative for $H \simeq T$).

N-1 Smeared Stringers

Read only if NSTRI = 1 on Card M-1.

Variable	Format	Columns	Description
E1	E 10.6	1-10	Young's modulus
U1	E 10.6	11-20	Poisson's ratio
ØI1	E 10.6	21-30	0.– Rectangular stringers 1.– Arbitrary stringers
D1	E 10.6	31-40	Stringer spacing (arc length)
AK1	E 10.6	41-50	0.– Internal stringers 1.– External stringers

N-2A Smeared Rectangular Stringers

Read only if ØI1 = 0 on Card N-1.

Variable	Format	Columns	Description
T1	E 10.6	1-10	Stringer thickness
H1	E 10.6	11-20	Stringer height

N-2B Smeared Arbitrary Stringers

Read only if ØI1 = 1 on Card N-1.

Variable	Format	Columns	Description
A1	E 10.6	1-10	Area of stringer
SI1	E 10.6	11-20	Moment of inertia of stringer about Y'-axis
XI1	E 10.6	21-30	Moment of inertia of stringer about Z'-axis
S1	E 10.6	31-40	Torsional stiffness of stringer, GJ
EZ1	E 10.6	41-50	Eccentricity of stringer. (See figure for discrete stringers.)
H1	E 10.6	51-60	Stringer Height

N-3 Smeared Rings

Read only if NRING = 1 on Card M-1.

Variable	Format	Columns	Description
E2	E 10.6	1–10	Young's Modulus
U2	E 10.6	11–20	Poisson's Ratio
ØI2	E 10.6	21–30	0.– Rectangular rings 1.– Arbitrary rings
D2	E 10.6	31–40	Ring spacing (arc length)
AK2	E 10.6	41–50	0.– Internal rings 1.– External rings

N-4A Smeared Rectangular Rings

Read only if ØI2 = 0 on Card N-3.

Variable	Format	Columns	Description
T2	E 10.6	1–10	Ring Thickness
H2	E 10.6	11–20	Ring Height

N-4B Smeared Arbitrary Rings

Read only if ØI2 = 1 on Card N-3.

Variable	Format	Columns	Description
A2	E 10.6	1–10	Area of ring
SI2	E 10.6	11–20	Moment of inertia of ring about X'-axis
XI2	E 10.6	21–30	Moment of inertia of ring about Z'-axis
S2	E 10.6	31–40	Torsional stiffness of ring, GJ
EZ2	E 10.6	41–50	Eccentricity of ring (See figure for discrete rings)
H2	E 10.6	51–60	Ring Height

Note: Stringers run parallel with X and rings parallel with Y coordinate axis.

The remaining cards are to be read only if IWALL = 9.

VARIABLE WALL PROPERTIES

V-1 (For Orthotropic Shell with variable properties – IWALL = 9)

Variable	Format	Columns	Description
TD	E 10.6	1–10	Wall thickness
Z	E 10.6	11–20	Distance from reference surface to midsurface of shell (positive if midsurface is outside of reference surface, i.e., at a positive Z coordinate)

V-2 X-Coarse Grid Cards

Variable	Format	Columns	Description
XM(I) I = 1,IM	8E 10.6	1–80	X-coordinates of selected mesh (for IM, see M-1 card)

Note: Card format is 8E 10.6; therefore, more than one V-2 card might be necessary to input all X coordinates.

V-3 Y-Coarse Grid Cards

Variable	Format	Columns	Description
YM(J) J = 1,JM	8E 10.6	1–80	Y-coordinates of selected mesh (for JM, see M-1 card)

Note: Card format is 8E 10.6; therefore, more than one V-3 card might be necessary to input all Y coordinates.

The following cards are read in a loop over the selected mesh points. They are read either by row or by column, depending on which direction has the largest number of original gridpoints. With NR (the number of points in the X-direction), NC (the number of points in the Y-direction – see Card D-1), and IM and JM (the number of selected points in these directions)

for NC ≤ NR	MSTA = JM NSTA = IM
for NC > NR	MSTA = IM NSTA = JM

The data card is repeated for L = 1, IP (number of points, numbered inner to outer, see M-1 card)

M = 1, MSTA

N = 1, NSTA

Note that L is the inner loop, M is the middle loop, and N is the outer loop.

V-4 Variable Properties Cards

Variable	Format	Columns	Description
TDEG(L, M, N)	E 10.6	1–10	Wall temperature
EX(L, M, N)	E 10.6	11–20	Modulus of elasticity in X direction
EY(L, M, N)	E 10.6	21–30	Modulus of elasticity in Y direction
U(L, M, N)	E 10.6	31–40	Poisson's ratio – μ_{xy}, ($\mu_{xy}E_x = \mu_{yx}E_y$)
G(L, M, N)	E 10.6	41–50	Shear modulus
A1(L, M, N)	E 10.6	51–60	Coefficient of thermal expansion in X direction
A2(L, M, N)	E 10.6	61–70	Coefficient of thermal expansion in Y direction

Note: Card format is 7E 10.6, therefore many V-4 cards will be necessary to input all wall properties.

END OF INPUT

Section 7
USE OF STAGS PROGRAM

This section illustrates the use of the STAGS computer program to solve some typical examples. It also illustrates the form and the interpretation of STAGS-generated output. To conserve space, only representative portions of the output are shown. Cases have been chosen which are deemed suitable for program checkout. Hence, together they cover most of the branches of the program, and they are relatively inexpensive to run. It is not intended to demonstrate the capability of the program; many of the cases can be solved by simpler means.

7.1 SAMPLE CASE 1 – CYLINDRICAL SHELL SEGMENT

Consider a circular cylindrical shell panel subjected to a radial load at the center (Fig. 7-1). The panel is simply supported along its curved boundaries and free along its straight boundaries. Because of the symmetrical arrangement, only one quarter of the panel need be analyzed. This is indicated in the figure; the boundaries are identified as shown. Figure 7-2 shows the input cards associated with this case; Fig. 7-3 shows portions of the output.

Computer output begins with a page containing pertinent input data and stiffness coefficients followed by the determinant of the system of equations and an indication of the elapsed time. The linear solution is printed in five segments (according to the O-1 type input card): first, the displacement field; second, the moment and stress resultants field; third, the stress field; fourth, selected displacements; fifth, selected moment and stress resultants. For cases in which the shell is loaded by a fixed displacement (unit end shortening), it is of interest to find the applied load. Therefore, the stress resultants are integrated at the NR-1 row and the result is given as output. In this particular case, the axial load, near zero, is of no interest.

After the linear solution, the program begins to compute the nonlinear displacements for successively increasing load factors as specified by means of the load increment parameter. Only the selected output related to load steps 1, 2, and 6 is shown. For each load step, the load factors PA and PB for load system A and load system B are printed. The load factor PA for this last step was 6.0, and the solution required 1 iteration. For each iteration, the following information is displayed: maximum displacement change, displacement component, location, relative error, and relaxation factor.

The nonlinear solution is displayed either in its entirety as in the linear solution or only at selected mesh points as requested by the user on the O-1 type of input card.

When program termination occurs because of an internal input card parameter (see L-1 and P-1B input cards), a statement describing the reason for termination will appear in the output and the last three successive solutions will be saved on tape 18 (output tape). A second run may start from any of these three records, provided changes are made to the input data, such as starting load factor (L-1 type input card) and restart command (P-1B type input card).

A display of input cards and selected output for the second run follows the first run (Figs. 7-3a and 7-3b).

Note that the load step increment may be cut or increased automatically based on an internal criteria.

The analysis may be continued until the collapse load has been determined, or convergence difficulties for small load increments are encountered.

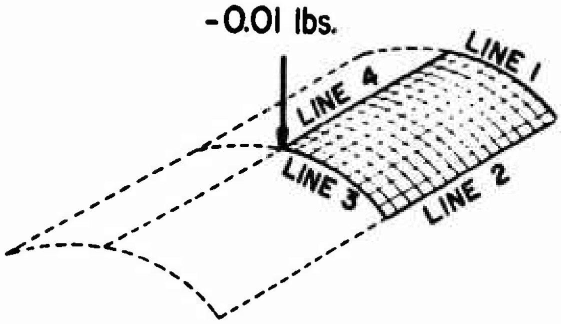


Fig. 7-1 Sample Case 1 – Cylindrical Shell Panel

SAMPLE CASE 1

FIRST RUN – INPUT

```
        SAMPLE CASE 1 - CYLINDRICAL SHELL PANEL              C-1
    1    2    1                                              G-1
3.0       22.5       2.5                                     G-2
   10    8                                                   D-1
    5    3    4    4                                         B-1
    1    0 1.      1.        6.                              L-1
-0.01     0.       0.             1    0    0   10    1      L-2
.001                    0   24    4    2    1                P-1B
    0    0   10    1    1    1    2                          O-1
 3.0                                                         O-2
 0.0                                                         O-3
    2                                                        M-1
0.01      10000000. 0.3                                      M-2B
```

Fig. 7-2 Display of Input Cards for Sample Case 1, First Run

SAMPLE CASE 1 FIRST RUN – OUTPUT

```
     SAMPLE CASE 1 - CYLINDRICAL SHELL PANEL
NON-LINEAR COLLAPSE ANALYSIS.          1 LOAD PATTERNS.

TYPE OF SURFACE IS CYLINDER

SURFACE CONSTANTS =  3.0000000E+00, 2.2500000E+01, 2.5000000E+00,

 BLANK COMMON ARRAY WORKING SPACE=     15000

  FINITE DIFFERENCE MESH.  10 ROWS.    8 COLUMNS.  MESH SPACING. H=    .3333, K=   3.2143

NRW1=  -0, NRW2= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS  ANTI-METRIC
BOUNDARY CONDITION AT LINE 2 IS UNRESTRAINED
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =     1

 USER-LOAD FLAG =      0.  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP =  1.0000000E+00,  MAXIMUM LOAD =  6.0000000E+00

        PZ              PY             PX          JZ   JY   JX  ROW  COL
-1.0000000E-02 0.                  0.               1    0    0   10    1

ERROR TOLERANCE =    1.0000000E-03 UNDERRELAXATION =   -0.
 ISTART   ISEC   ICUT   INENT   ISTRAT
     0     24      4      2      1

IPX=     1 IPY=     1 IPRD=     1 IPRS=     2 IPLOT=     -0

IWALL=    2,  NSTRI=   -0,  NRING=   -0,  IP=   -0,  IM=   -0,  JM=   -0

  AT =  1.0000000E-02  EX1 =  1.0000000E+07 XNU =  3.0000000E-01
   Z = -0.             EY1 =  1.0000000E+07   G =  3.8461538E+06


THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CF02
      CCC(I,1)            CCC(I,2)            CCC(I,3)            CCC(I,4)            CCC(I,5)            CCC(I,6)

     1.098901E+05        0.                  0.                  0.                  0.                  0.
     3.296703E+04        1.098901E+05        0.                  0.                  0.                  0.
     0.                  0.                  3.846154E+04        0.                  0.                  0.
     0.                  0.                  0.                  9.157509E-01        0.                  0.
     0.                  0.                  0.                  2.747253E-01        9.157509E-01        0.
     0.                  0.                  0.                  0.                  0.                  3.205128E-01
```

Fig. 7-3 Excerpt of Output for Sample Case 1, First Run.

```
CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=    1.157.  NR OF IO REQUESTS (TAPE2)=    11.      WORDS USED (TAPE2)=   28532.     WORDS TRANSFERRED (TAPE2)=        30588

 FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=    2.224.  NR OF IO REQUESTS (TAPE2)=    23.      WORDS USED (TAPE2)=   49291.     WORDS TRANSFERRED (TAPE2)=        74146

 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=    2.595.  NR OF IO REQUESTS (TAPE2)=    29.      WORDS USED (TAPE2)=   63301.     WORDS TRANSFERRED (TAPE2)=       108915


DETERMINANT OF STIFFNESS MATRIX=   1.5754282E+09*10.**   458.   NUMBER OF NEGATIVE ROOTS =   0
        80 NODES.        360 EQUATIONS.   MAXIMUM BAND WIDTH =          66
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=    3.758.  NR OF IO REQUESTS (TAPE2)=    35.      WORDS USED (TAPE2)=   63301.     WORDS TRANSFERRED (TAPE2)=       137959



LINEAR SOLUTION.  PA=   1.000000E+00,    PB=   0.
ROW  9.   X=      2.667, AXIAL LOAD=       -5.146594E-13,BENDING MOMENT=      0.

  ROW    1.   X=      0.0000
                                   OUTER SURFACE                                          INNER SURFACE
     X          Y        SIGMAX        SIGMAY          TAU          SIGMAX        SIGMAY           TAU     SIGMA(STRNGR)   SIGMA(RING)
   0.000     0.000    3.5814E-14    1.1938E-13    0.               3.5671E-14    1.1890E-13    0.
   0.000     3.214    7.3897E-13    6.5624E-13    1.0033E+00       7.3840E-13    6.5433E-13    3.5129E+00
   0.000     6.429    8.8223E-13    1.1338E-12    1.1128E+00       8.8109E-13    1.1300E-12    5.9524E+00
   0.000     9.643    1.4893E-13    4.4679E-14    2.5598E+00       1.4893E-13    4.4679E-14    9.2542E+00
   0.000    12.857    0.            0.            2.5339E+00       0.            0.            1.0598E+01
   0.000    16.071   -5.9572E-13   -1.7871E-13    2.2624E+00      -5.9572E-13   -1.7871E-13    1.1185E+01
   0.000    19.286    0.            0.            2.7480E-01       0.            0.            9.4733E+00
   0.000    22.500   -2.3829E-12   -7.1486E-13   -1.2860E+00      -2.3829E-12   -7.1486E-13    7.8407E+00

  ROW    2.   X=       .3333
                                   OUTER SURFACE                                          INNER SURFACE
     X          Y        SIGMAX        SIGMAY          TAU          SIGMAX        SIGMAY           TAU     SIGMA(STRNGR)   SIGMA(RING)
    .333     0.000   -5.9328E+00   -4.1225E+00    0.              -3.5174E+00    3.8902E+00    0.
    .333     3.214   -6.1041E+00   -4.0072E+00    3.2184E-01      -3.3670E+00    4.2259E+00    3.1231E+00
    .333     6.429   -5.8501E+00   -3.5045E+00    1.6866E+00      -3.1007E+00    3.5078E+00    6.9601E+00
    .333     9.643   -4.9005E+00   -2.7793E+00    1.6302E+00      -2.0577E+00    2.6306E+00    8.9617E+00
    .333    12.857   -3.4352E+00   -2.0492E+00    2.4678E+00      -1.3942E-01    2.1728E+00    1.1238E+01
    .333    16.071   -4.5932E-02   -1.2710E+00    1.9225E+00       3.4426E+00    1.1617E+00    1.1523E+01
    .333    19.286    5.3671E+00   -5.8245E-01   -1.7245E-01       9.2429E+00    5.4902E-01    9.7606E+00
    .333    22.500    1.3415E+01   -2.7714E-01   -1.6204E+00       1.7932E+01    3.1085E-01    8.2548E+00

  ROW    3.   X=       .6667
                                   OUTER SURFACE                                          INNER SURFACE
     X          Y        SIGMAX        SIGMAY          TAU          SIGMAX        SIGMAY           TAU     SIGMA(STRNGR)   SIGMA(RING)
    .667     0.000   -1.2787E+01   -9.2843E+00    0.              -6.6988E+00    1.0003E+01    0.
    .667     3.214   -1.2210E+01   -8.7310E+00    6.0288E-01      -6.6870E+00    8.4513E+00    4.1407E+00
    .667     6.429   -1.1658E+01   -7.6018E+00    1.7223E-01      -5.8011E+00    7.6457E+00    6.9151E+00
    .667     9.643   -1.0489E+01   -6.0436E+00    1.3994E+00      -4.1615E+00    6.3185E+00    1.0560E+01
    .667    12.857   -5.3991E+00   -4.1745E+00    1.2645E+00      -1.2696E-01    3.7600E+00    1.2059E+01
    .667    16.071   -5.3265E-02   -2.5909E+00    8.0936E-01       6.9840E+00    2.6134E+00    1.2529E+01
    .667    19.286    1.0691E+01   -1.2422E+00   -1.1990E+00       1.8402E+01    1.2209E+00    1.0814E+01
    .667    22.500    2.6909E+01   -5.8901E-01   -2.7236E+00       3.5594E+01    4.8862E-01    9.1982E+00
```

Fig. 7-3 (Cont.)

ROW 8, X= 2.3333

X	Y	OUTER SURFACE SIGMAX	OUTER SURFACE SIGMAY	TAU	INNER SURFACE SIGMAX	INNER SURFACE SIGMAY	TAU	SIGMA(STRNGR)	SIGMA(RING)
2.333	0.000	-6.1421E+01	-7.8538E+01	0.	-2.1033E+01	7.5612E+01	0.		
2.333	3.214	-6.5328E+01	-7.2056E+01	-1.0020E+01	-1.6755E+01	7.9124E+01	1.1946E+01		
2.333	6.429	-6.3390E+01	-5.1504E+01	-5.3685E+00	-1.5374E+01	5.9969E+01	2.3587E+01		
2.333	9.643	-3.7815E+01	-2.4658E+01	1.2108E+00	-1.0005E+01	2.2090E+01	2.5315E+01		
2.333	12.857	-3.2332E+00	-4.5733E+00	2.2110E+00	5.2278E+00	-1.7406E-01	1.9152E+01		
2.333	16.071	3.0680E+01	2.9254E+00	-1.0925E+00	2.7431E+01	-6.7818E+00	1.1692E+01		
2.333	19.286	6.0277E+01	2.8836E+00	-5.2167E+00	5.1935E+01	-4.1992E+00	6.5408E+00		
2.333	22.500	8.3539E+01	1.5594E+00	-7.0775E+00	7.1314E+01	-1.7260E+00	5.0532E+00		

ROW 9, X= 2.6667

X	Y	OUTER SURFACE SIGMAX	OUTER SURFACE SIGMAY	TAU	INNER SURFACE SIGMAX	INNER SURFACE SIGMAY	TAU	SIGMA(STRNGR)	SIGMA(RING)
2.667	0.000	-1.0547E+02	-1.3640E+02	0.	-2.3873E+01	1.7123E+02	0.		
2.667	3.214	-9.8966E+01	-9.9608E+01	-3.1412E+00	-2.4695E+01	1.1048E+02	3.8225E+01		
2.667	6.429	-5.9346E+01	-4.1381E+01	4.9053E+00	-2.7161E+01	2.7786E+01	3.4681E+01		
2.667	9.643	-1.5704E+01	-4.4069E+00	6.7776E+00	-1.3712E+01	-7.4583E+00	2.0265E+01		
2.667	12.857	1.9665E+01	7.6911E+00	4.0110E+00	8.7638E+00	-1.3064E+01	9.3331E+00		
2.667	16.071	4.7034E+01	7.0141E+00	-4.3708E-02	3.3857E+01	-8.0907E+00	3.4436E+00		
2.667	19.286	6.7803E+01	3.2363E+00	-2.4637E+00	5.4211E+01	-2.8287E+00	1.4285E+00		
2.667	22.500	8.0816E+01	1.4034E+00	-3.0893E+00	6.5017E+01	-1.1486E+00	1.2764E+00		

ROW 10, X= 3.0000

X	Y	OUTER SURFACE SIGMAX	OUTER SURFACE SIGMAY	TAU	INNER SURFACE SIGMAX	INNER SURFACE SIGMAY	TAU	SIGMA(STRNGR)	SIGMA(RING)
3.000	0.000	-2.6897E+02	-4.5446E+02	0.	7.9131E+01	3.2235E+02	0.		
3.000	3.214	-1.1780E+02	-1.1865E+02	8.3567E-13	-2.8743E+01	2.4769E+01	8.3233E-13		
3.000	6.429	-3.3556E+01	-4.5910E+00	5.4619E-12	-4.0471E+01	-3.3634E+01	4.5878E-12		
3.000	9.643	6.6657E+00	1.7261E+01	1.7026E-12	-1.8459E+01	-2.9253E+01	1.6959E-12		
3.000	12.857	3.1721E+01	1.7786E+01	3.5979E-12	1.1550E+01	-1.3960E+01	3.1575E-12		
3.000	15.071	5.2384E+01	6.7233E+00	1.7547E-12	3.7199E+01	-5.4632E+00	1.7481E-12		
3.000	19.286	6.8780E+01	2.5972E+00	3.5028E-12	5.5086E+01	-1.8112E+00	3.1692E-12		
3.000	22.500	7.7924E+01	1.1715E+00	4.0032E-14	6.3630E+01	-6.6532E-01	-4.0032E-14		

CP SECONDS= 4.512. NR OF IO REQUESTS (TAPE2)= 54. WORDS USED (TAPE2)= 63513. WORDS TRANSFERRED (TAPE2)= 223299

ROW 10, X= 3.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-4.522421E-04	0.	-1.694066E-21	0.	0.
2	3.2143	-3.954016E-04	2.361834E-05	-3.388132E-21	0.	5.533350E-04
3	6.4286	-3.043320E-04	4.301637E-05	0.	0.	6.161943E-04
4	9.6429	-2.177331E-04	5.757618E-05	0.	0.	5.698869E-04
5	12.8571	-1.380691E-04	6.747871E-05	8.470329E-22	0.	5.276563E-04
6	16.0714	-6.215477E-05	7.295713E-05	8.470329E-22	0.	5.074031E-04
7	19.2857	1.244292E-05	7.413422E-05	1.694066E-21	0.	5.018372E-04
8	22.5000	8.692099E-05	7.107224E-05	3.388132E-21	0.	5.033521E-04

COL 1, Y= 0.0000

ROW	X	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	8.667979E-06	-1.526515E-04	0.
2	.3333	-5.088364E-05	0.	8.589808E-06	-1.526322E-04	0.
3	.6667	-1.017549E-04	0.	8.347456E-06	-1.521087E-04	0.
4	1.0000	-1.522495E-04	0.	7.943465E-06	-1.514501E-04	0.
5	1.3333	-2.027215E-04	0.	7.354574E-06	-1.494263E-04	0.
6	1.6667	-2.519372E-04	0.	6.575903E-06	-1.470156E-04	0.
7	2.0000	-3.007319E-04	0.	5.543834E-06	-1.415529E-04	0.
8	2.3333	-3.462758E-04	0.	4.261475E-06	-1.463788E-04	0.
9	2.6667	-3.993178E-04	0.	2.416683E-06	-1.739495E-04	0.
10	3.0000	-4.622421E-04	0.	-1.694066E-21	0.	0.

Fig. 7-3 (Cont.)

ROW 10, X= 3.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-9.492186E-01	-6.605277E-01	0.	-2.900878E-03	-6.473378E-03	0.
2	3.2143	-7.327050E-01	-4.693025E-01	8.340017E-15	-7.421298E-04	-1.195119E-03	2.780006E-20
3	6.4286	-3.716373E-01	-1.911235E-01	5.024960E-14	5.512070E-05	2.420232E-04	7.203615E-18
4	9.6429	-5.895481E-02	-5.995800E-02	1.699278E-14	2.093700E-04	3.076214E-04	5.569911E-20
5	12.8571	2.168541E-01	-3.474569E-03	3.377707E-14	1.672574E-04	2.272165E-04	3.669607E-18
6	16.0714	4.479173E-01	6.330448E-03	1.751404E-14	1.265419E-04	1.016037E-04	5.560011E-20
7	19.2857	6.193343E-01	3.930393E-03	3.336007E-14	1.141163E-04	3.673671E-05	2.780006E-18
8	22.5000	7.076229E-01	2.531126E-03	0.	1.193699E-04	1.530718E-05	6.672013E-19

COL 1, Y= 0.0000

ROW	X	NX	NY	NXY	MX	MY	MXY
1	0.0000	3.574293E-16	1.191431E-15	0.	1.191431E-21	3.971437E-21	0.
2	.3333	-4.725958E-02	-1.161191E-02	0.	-2.012836E-05	-6.677226E-05	0.
3	.6667	-9.743151E-02	3.593326E-03	0.	-5.073089E-05	-1.607268E-04	0.
4	1.0000	-1.447327E-01	-2.826934E-03	0.	-7.082753E-05	-2.601046E-04	0.
5	1.3333	-2.058427E-01	1.202593E-02	0.	-1.439092E-04	-4.485415E-04	0.
6	1.6667	-2.592271E-01	-4.917893E-03	0.	-1.887745E-04	-6.202251E-04	0.
7	2.0000	-3.509232E-01	3.520229E-02	0.	-3.256276E-04	-1.003404E-03	0.
8	2.3333	-4.122731E-01	-1.448181E-02	0.	-3.365633E-04	-1.284331E-03	0.
9	2.6667	-6.466376E-01	1.741729E-01	0.	-6.795540E-04	-2.563571E-03	0.
10	3.0000	-9.492186E-01	-6.605277E-01	0.	-2.900878E-03	-6.473378E-03	0.

BEGIN NON-LINEAR ITERATION FOR LOAD STEP 1, PA= 1.0000E+00, PB= 0.

ITERATION	MAXIMUM DISPLACEMENT CHANGE	COMPONENT	ROW	COL	REL ERROR	RELAXATION FACTOR
1	1.575979E-06	W	10	1	3.397838E-03	1.0000
2	-1.619192E-06	W	10	9	3.408766E-03	1.0000
3	1.895951E-08	W	10	9	4.084852E-05	1.0000

LOAD STEP 1, PA=1.0000E+00, PB=0. , 4 ITERATIONS. RELATIVE ERROR= 4.084852E-05
CP SECONDS= 5.715. NR OF IO REQUESTS (TAPE2)= 81. WORDS USED (TAPE2)= 63813. WORDS TRANSFERRED (TAPE2)= 364167
ROW 9, X= 2.667, AXIAL LOAD= -2.243245E-07, BENDING MOMENT= 0.

ROW 10, X= 3.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-4.641173E-04	0.	-1.694066E-21	0.	0.
2	3.2143	-3.970401E-04	2.370422E-05	-2.541099E-21	0.	5.558051E-04
3	6.4286	-3.055323E-04	4.319561E-05	-1.694066E-21	0.	6.194722E-04
4	9.6429	-2.184365E-04	5.774242E-05	-2.117582E-22	0.	5.726215E-04
5	12.8571	-1.384337E-04	6.765347E-05	8.470329E-22	0.	5.297091E-04
6	16.0714	-6.226276E-05	7.312601E-05	1.694066E-21	0.	5.098040E-04
7	19.2857	1.254635E-05	7.428477E-05	1.694066E-21	0.	5.032257E-04
8	22.5000	8.722644E-05	7.119265E-05	3.388132E-21	0.	5.046717E-04

COL 1, Y= 0.0000

ROW	X	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	8.707740E-06	-1.533322E-04	0.
2	.3333	-5.111075E-05	0.	8.627330E-06	-1.532985E-04	0.
3	.6667	-1.021990E-04	0.	8.380128E-06	-1.527251E-04	0.
4	1.0000	-1.529275E-04	0.	7.970895E-06	-1.519995E-04	0.
5	1.3333	-2.035328E-04	0.	7.376542E-06	-1.499042E-04	0.
6	1.6667	-2.528636E-04	0.	6.592656E-06	-1.474564E-04	0.
7	2.0000	-3.018363E-04	0.	5.555787E-06	-1.419785E-04	0.
8	2.3333	-3.475168E-04	0.	4.269362E-06	-1.469958E-04	0.
9	2.6667	-3.998335E-04	0.	2.420273E-06	-1.749028E-04	0.
10	3.0000	-4.641173E-04	0.	-1.694066E-21	0.	0.

Fig. 7-3 (Cont.)

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BEGIN NON-LINEAR ITERATION FOR LOAD STEP   2.  PA= 2.0000E+00,  PB= 0.

ITERATION  MAXIMUM DISPLACEMENT CHANGE    COMPONENT     ROW   COL       REL ERROR    RELAXATION FACTOR
    0               5.446691E-06              W          10     9    5.851333E-03               1.0000
    1              -6.662290E-06              W          10     9    7.169412E-03               1.0000
    2               3.581321E-07              W          10     9    3.842457E-04                .0000

 LOAD STEP  2,  PA=2.0000E+00,  PB=0.         .    3 ITERATIONS.  RELATIVE ERROR=    3.842457E-04
CP SECONDS=     6.911.  NR OF IO REQUESTS (TAPE2)=  114.     WORDS USED (TAPE2)=  64325.    WORDS TRANSFERRED (TAPE2)=      520581
ROW  9.  X=     2.667, AXIAL LOAD=       3.385475E-05,BENDING MOMENT=     0.

  ROW  10.  X=     3.0000
  COL       Y          W                V               U             BETAX           BETAY
    1     0.0000   -9.320394E-04    0.              -3.388132E-21   0.              0.
    2     3.2143   -7.974050E-04    4.758278E-05    -3.388132E-21   0.              1.116927E-03
    3     6.4286   -6.134033E-04    8.659375E-05    -1.694066E-21   0.              1.245568E-03
    4     9.6429   -4.383092E-04    1.158224E-04    -4.235165E-22   0.              1.150759E-03
    5    12.8571   -2.776209E-04    1.356625E-04     1.694066E-21   0.              1.063546E-03
    6    16.0714   -1.247641E-04    1.465968E-04     1.694066E-21   0.              1.021213E-03
    7    19.2857    2.527639E-05    1.482787E-04     6.776264E-21   0.              1.009221E-03
    8    22.5000    1.750260E-04    1.426353E-04     3.388132E-21   0.              1.011970E-03

  COL   1.  Y=     0.0000
  ROW       X          W                V               U             BETAX           BETAY
    1     0.0000   0.               0.               1.749607E-05   -3.080361E-04   0.
    2      .3333   -1.026787E-04    0.               1.733957E-05   -3.079406E-04   0.
    3      .6667   -2.052337E-04    0.               1.682631E-05   -3.066945E-04   0.
    4     1.0000   -3.071417E-04    0.               1.599726E-05   -3.051100E-04   0.
    5     1.3333   -4.087004E-04    0.               1.479754E-05   -3.007773E-04   0.
    6     1.6667   -5.076599E-04    0.               1.321928E-05   -2.958093E-04   0.
    7     2.0000   -6.059366E-04    0.               1.113587E-05   -2.848240E-04   0.
    8     2.3333   -6.975431E-04    0.               8.554802E-06   -2.952507E-04   0.
    9     2.6667   -8.027434E-04    0.               4.847869E-06   -3.517444E-04   0.
   10     3.0000   -9.320394E-04    0.              -3.388132E-21   0.              0.

  ROW  10.  X=     3.0000
  COL       Y          NX               NY              NXY             MX              MY              MXY
    1     0.0000   -1.903391E+00   -1.329754E+00    0.              -5.851944E-03   -1.384153E-02   0.
    2     3.2143   -1.469739E+00   -9.433687E-01    5.087410E-14    -1.514759E-03   -2.441562E-03   1.668003E-19
    3     6.4286   -7.471051E-01   -3.832539E-01    6.713713E-14     1.079558E-04    4.908090E-04   1.445603E-17
    4     9.6429   -1.293625E-01   -1.208271E-01    3.419407E-14     4.253868E-04    7.958638E-04   1.434483E-17
    5    12.8571    4.329265E-01   -7.329808E-03    3.419407E-14     3.395726E-04    4.700305E-04   7.222014E-18
    6    16.0714    8.973742E-01    1.238029E-02    3.336007E-14     2.542636E-04    2.111761E-04   3.669607E-18
    7    19.2857    1.244490E+00    7.839121E-03    3.336037E-14     2.258736E-04    7.537292E-05   1.890404E-18
    8    22.5000    1.428642E+00    5.155367E-03    3.336007E-14     2.339596E-04    2.991075E-05   2.112804E-18

  COL   1.  Y=     0.0000
  ROW       X          NX               NY              NXY             MX              MY              MXY
    1     0.0000    5.213531E-03    1.564955E-03    0.              0.              0.              0.
    2      .3333   -9.523537E-02   -2.269662E-03    0.              -4.234487E-05   -1.395542E-04   0.
    3      .6667   -1.963298E-01    7.468543E-03    0.              -1.057558E-04   -3.333420E-04   0.
    4     1.0000   -2.912113E-01   -5.648657E-03    0.              -1.627952E-04   -5.354175E-04   0.
    5     1.3333   -4.137737E-01    2.452969E-02    0.              -2.943737E-04   -9.162659E-04   0.
    6     1.6667   -5.199201E-01   -9.968801E-03    0.              -3.831780E-04   -1.259441E-03   0.
    7     2.0000   -7.032358E-01    7.148814E-02    0.              -6.590901E-04   -2.031712E-03   0.
    8     2.3333   -8.241206E-01   -2.858667E-02    0.              -6.755121E-04   -2.590727E-03   0.
    9     2.6667   -1.293031E+00    3.501327E-01    0.              -1.371077E-03   -5.172799E-03   0.
   10     3.0000   -1.903391E+00   -1.329754E+00    0.              -5.851944E-03   -1.384153E-02   0.
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Fig. 7-3 (Cont.)

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BEGIN NON-LINEAR ITERATION FOR LOAD STEP   6,  PA= 6.0000E+00,  PB= 0.

ITERATION  MAXIMUM DISPLACEMENT CHANGE    COMPONENT     ROW   COL        REL ERROR    RELAXATION FACTOR
    3              1.182051E-06               W          10     9     4.155641E-04              .9650


 LOAD STEP  5.  PA=6.0000E+00,  PB=0.                1 ITERATIONS.  RELATIVE ERROR=     4.155641E-04
CP SECONDS=    9.812.  NR OF IO REQUESTS (TAPE2)=   216.       WORDS USED (TAPE2)=   65349.     WORDS TRANSFERRED (TAPE2)=      914211

MAXIMUM LOAD VALUE ATTAINED

NONLINEAR COLLAPSE ANALYSIS DISCONTINUED
 DISPLACEMENT SOLUTION FOR PA=4.000000E+00,  PB=0.              WRITTEN ON TAPE18.
 DISPLACEMENT SOLUTION FOR PA=5.000000E+00,  PB=0.              WRITTEN ON TAPE18.
 DISPLACEMENT SOLUTION FOR PA=6.000000E+00,  PB=0.              WRITTEN ON TAPE18.
ROW  9.  X=      2.667, AXIAL LOAD=      -9.385305E-05,BENDING MOMENT=      0.

  ROW  10.  X=     3.0000
  COL         Y            W                 V                  U                BETAX                 BETAY
    1      0.0000    -2.844449E-03    0.                   -6.776264E-21    0.                    0.
    2      3.2143    -2.434517E-03    1.449662E-04         -1.355253E-20    0.                    3.416004E-03
    3      6.4286    -1.660997E-03    2.633801E-04          0.              0.                    3.821233E-03
    4      9.6429    -1.333113E-03    3.517648E-04         -1.694066E-21    0.                    3.523937E-03
    5     12.8571    -8.420670E-04    4.115028E-04          3.3[illegible]   0.                    3.245740E-03
    6     16.0714    -3.765143E-04    4.441401E-04          1.355253E-20    0.                    3.107875E-03
    7     19.2857     7.950922E-05    4.504796E-04          6.776264E-21    0.                    3.066902E-03
    8     22.5000     5.342653E-04    4.309151E-04          1.355253E-20    0.                    3.073485E-03

  COL   1,  Y=     0.0000
  ROW         X            W                 V                  U                BETAX                 BETAY
    1      0.0000     0.              0.                    5.349192E-05   -9.411945E-04    0.
    2       .3333    -3.137315E-04    0.                    5.293764E-05   -9.405980E-04    0.
    3       .6667    -6.270654E-04    0.                    5.130283E-05   -9.357025E-04    0.
    4      1.0000    -9.375332E-04    0.                    4.868511E-05   -9.293820E-04    0.
    5      1.3333    -1.246653E-03    0.                    4.495086E-05   -9.147197E-04    0.
    6      1.6667    -1.547346E-03    0.                    4.00873[illegible]E-05   -8.990106E-04    0.
    7      2.0000    -1.845994E-03    0.                    3.371786E-05   -8.657464E-04    0.
    8      2.3333    -2.124511E-03    0.                    2.587152E-05   -9.019863E-04    0.
    9      2.6667    -2.447318E-03    0.                    1.463843E-05   -1.079907E-03    0.
   10      3.0000    -2.844449E-03    0.                   -6.776264E-21    0.               0.

  ROW  10.  Y=     3.0000
  COL         Y            NX               NY               NXY              MX               MY               MXY
    1      0.0000    -5.742068E+00    -4.043823E+00    0.               -1.787445E-02    -3.972496E-02    0.
    2      3.2143    -4.437841E+00    -2.859457E+00    6.672013E-14     -4.739820E-03    -7.653640E-03    2.224004E-19
    3      6.4286    -2.266221E+00    -1.157210E+00    6.838814E-14      3.056110E-04     1.507813E-03    2.224004E-19
    4      9.6429    -3.764895E-01    -3.698495E-01    2.677145E-13      1.315418E-03     2.511434E-03    8.896018E-19
    5     12.8571     1.294615E+00    -2.617921E-02    0.                1.049824E-03     1.504114E-03    2.846726E-17
    6     16.0714     2.702784E+00     3.355583E-02    0.                7.711028E-04     6.810842E-04    1.423363E-17
    7     19.2857     3.759587E+00     2.138561E-02    0.                6.652810E-04     2.373308E-04    7.116814E-18
    8     22.5000     4.367589E+00     1.388920E-02    0.                6.756565E-04     8.552649E-05    1.156482E-17

  COL   1.  Y=     0.0000
  ROW         X            NX               NY               NXY              MX               MY               MXY
    1      0.0000     4.867291E-02     1.460187E-02    0.                0.               0.               0.
    2       .3333    -2.900529E-01    -6.374166E-03    0.               -1.396648E-04    -4.556090E-04    0.
    3       .6667    -5.976359E-01     2.410532E-02    0.               -3.435520E-04    -1.073522E-03    0.
    4      1.0000    -8.838079E-01    -1.689354E-02    0.               -5.199420E-04    -1.699449E-03    0.
    5      1.3333    -1.253712E+00     7.662244E-02    0.               -9.237345E-04    -2.868434E-03    0.
    6      1.6667    -1.568392E+00    -3.071201E-02    0.               -1.184534E-03    -3.897311E-03    0.
    7      2.0000    -2.119213E+00     2.213157E-01    0.               -2.026773E-03    -6.252543E-03    0.
    8      2.3333    -2.470326E+00    -8.327561E-02    0.               -2.041607E-03    -7.912623E-03    0.
    9      2.6667    -3.877425E+00     1.061329E+00    0.               -4.165786E-03    -1.581071E-02    0.
   10      3.0000    -5.742068E+00    -4.043823E+00    0.               -1.787445E-02    -3.972496E-02    0.
CP SECONDS=   10.075.  NR OF IO REQUESTS (TAPE2)=   226.       WORDS USED (TAPE2)=   65349.     WORDS TRANSFERRED (TAPE2)=      950516
```

Fig. 7-3 (Cont.)

The second nonlinear run is a continuation of the first run, which terminated (because the maximum load was obtained) at load step 6 with PA equal 6.0. Because the run was started with the 3rd record, i.e. ISTART equal 3 (see p-1B type input card), the starting load factor must be changed accordingly. Hence, STLD is set equal 6.0 (see L-1 type input card).

SAMPLE CASE 1
SECOND RUN – INPUT

```
      SAMPLE CASE 1 - CYLINDRICAL SHELL PANEL              C-1
    1    2    1                                            G-1
3.0       22.5      2.5                                    G-2
   10    8                                                 D-1
    5    3    4    4                                       B-1
    1    0 6.       1.        8.                           L-1
-0.01     0.        0.        1    0    0   10    1        L-2
.001                3   24    4    2    1                  P-1B
    3    0   10    1    1    1    2                        O-1
 3.0                                                       O-2
 0.0                                                       O-3
    2                                                      M-1
0.01      10000000. 0.3                                    M-2B
```

Fig. 7-3a Display of Input Cards for Sample Case 1, Second Run

SAMPLE CASE 1, SECOND RUN – OUTPUT

```
      SAMPLE CASE 1 - CYLINDRICAL SHELL PANEL
NON-LINEAR COLLAPSE ANALYSIS.           1 LOAD PATTERNS.

TYPE OF SURFACE IS CYLINDER

SURFACE CONSTANTS =  3.0000000E+00, 2.2500000E+01, 2.5000000E+00,

 BLANK COMMON ARRAY WORKING SPACE=     15000

  FINITE DIFFERENCE MESH.  10 ROWS,    5 COLUMNS.  MESH SPACING. H=    .3333, K=   3.2143

NRW1=  -0, NRW2= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS  ANTI-METRIC
BOUNDARY CONDITION AT LINE 2 IS UNRESTRAINED
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =     1

 USER-LOAD FLAG =     0,  STARTING LOAD FACTOR =  6.0000000E+00,  LOAD STEP =  1.0000000E+00,  MAXIMUM LOAD =  8.0000000E+00

      PZ              PY              PX          JZ   JY   JX  ROW  COL
-1.0000000E-02 0.              0.                  1    0    0   10    1

ERROR TOLERANCE =    1.000000E-03 UNDERRELAXATION =    -0.
 ISTART    ISEC    ICUT   INEWT   ISTRAT
     3      24       4       2       1

IPX=      1 IPY=     1 IPRO=     1 IPRS=     2 IPLOT=    -0

IWALL=      2,  NSTRI=   -0,  NRING=   -0,  IP=   -0,  IM=   -0,  JM=,  -0

  AT =  1.0000000E-02  EX1 =  1.0000000E+07 XNU =  3.0000000E-01
   Z = -0.             EY1 =  1.0000000E+07   G =  3.8461538E+06


THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CF02
     CCC(I,1)            CCC(I,2)            CCC(I,3)            CCC(I,4)            CCC(I,5)            CCC(I,6)

       1.098901E+05        0.                  0.                  0.                  0.                  0.
       3.296703E+04        1.098901E+05        0.                  0.                  0.                  0.
       0.                  0.                  3.846154E+04        0.                  0.                  0.
       0.                  0.                  0.                  9.157509E-01        0.                  0.
       0.                  0.                  0.                  2.747253E-01        9.157509E-01        0.
       0.                  0.                  0.                  0.                  0.                  3.205128E-01
```

Fig. 7-3b Excerpt of Output for Sample Case 1, Second Run

```
CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=    1.183.  NR OF IO REQUESTS (TAPE2)=    11. .    WORDS USED (TAPE2)=   28532.    WORDS TRANSFERRED (TAPE2)=       38588

FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=    2.939.  NR OF IO REQUESTS (TAPE2)=    27.      WORDS USED (TAPE2)=   58827.    WORDS TRANSFERRED (TAPE2)=       76196

ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=    3.273.  NR OF IO REQUESTS (TAPE2)=    33.      WORDS USED (TAPE2)=   64837.    WORDS TRANSFERRED (TAPE2)=      110963


DETERMINANT OF STIFFNESS MATRIX=   1.6923785E+09*10.**   458.   NUMBER OF NEGATIVE ROOTS =   0
      80 NODES,        360 EQUATIONS.   MAXIMUM BAND WIDTH =          66
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=    4.426.  NR OF IO REQUESTS (TAPE2)=    39.      WORDS USED (TAPE2)=   64537.    WORDS TRANSFERRED (TAPE2)=      140887



BEGIN NON-LINEAR ITERATION FOR LOAD STEP   7,  PA= 7.0000E+00,  PB= 0.

ITERATION   MAXIMUM DISPLACEMENT CHANGE    COMPONENT     ROW   COL        REL ERROR    RELAXATION FACTOR
    3               -9.019300E-07              W          10     9     2.705598E-04              1.0000

 LOAD STEP  7,  PA=7.0000E+00,  PB=0.        ,    1 ITERATIONS.  RELATIVE ERROR=     2.705598E-04
CP SECONDS=    4.806.  NR OF IO REQUESTS (TAPE2)=    54.      WORDS USED (TAPE2)=   65349.    WORDS TRANSFERRED (TAPE2)=      185877
ROW  9,  X=      2.667, AXIAL LOAD=        1.542691E-05,BENDING MOMENT=     0.

  ROW  13,  X=     3.0000
  COL        Y            W                 V                U               BETAX            BETAY
    1     0.0000   -3.333459E-03    0.                0.               0.               0.
    2     3.2143   -2.853351E-03    1.698141E-04     -6.776264E-21     0.               4.805437E-03
    3     6.4286   -2.193369E-03    3.093913E-04      0.               0.               4.484489E-03
    4     9.6429   -1.560453E-03    4.117225E-04     -1.694066E-21     0.               4.134066E-03
    5    12.8571   -9.850711E-04    4.814793E-04      6.776264E-21     0.               3.804722E-03
    6    16.0714   -4.396392E-04    5.194927E-04      6.776264E-21     0.               3.640681E-03
    7    19.2857    9.442454E-05    5.267115E-04      1.355253E-20     0.               3.591397E-03
    8    22.5000    6.265723E-04    5.035979E-04      2.710505E-20     0.               3.598779E-03

  COL    1,  Y=    3.0000
  ROW        X            W                 V                U               BETAX            BETAY
    1     0.0000    0.               0.                6.271114E-05    -1.183238E-03    0.
    2      .3333   -3.677460E-04     0.                6.294741E-05    -1.192461E-03    0.
    3      .6667   -7.349740E-04     0.                6.013306E-05    -1.096424E-03    0.
    4     1.0000   -1.093695E-03     0.                5.703997E-05    -1.088604E-03    0.
    5     1.3333   -1.469710E-03     0.                5.261327E-05    -1.071027E-03    0.
    6     1.6667   -1.912713E-03     0.                4.690360E-05    -1.052404E-03    0.
    7     2.0000   -2.162366E-03     0.                3.943385E-05    -1.013597E-03    0.
    8     2.3333   -2.488444E-03     0.                3.024034E-05    -1.057407E-03    0.
    9     2.6667   -2.967334E-03     0.                1.710303E-05    -1.267522E-03    0.
   10     3.0000   -3.333459E-03     0.                0.               0.              0.
```

Fig. 7-3b (Cont.)

7.2 SAMPLE CASE 2 – CYLINDRICAL SHELL WITH ELLIPTIC CROSS SECTION

This case demonstrates calculation of the buckling load, according to bifurcation theory, for a cylinder with elliptic cross section (Fig. 7-4) and subjected to the combined effects of a line load along boundary line 1 (load pattern A) and uniform internal pressure (load pattern B). Here will be determined the critical axial load corresponding to an internal pressure of 20 psi. Because of symmetry, only one eighth of the cylinder needs to be analyzed. In the prebuckling analysis, the restricting assumption is made that the buckling pattern also is symmetric about the same two planes. The input cards associated with this case are illustrated in Fig. 7-5. Portions of the output are displayed in Fig. 7-6.

After displaying the input parameters, stiffness coefficients, and the determinant of the system, the linear solution is given as in the previous example, followed by the determinant of the system based on the new boundary conditions (if the incremental and prebuckling displacement boundary conditions are different) and an initial shift if any. The eigenvalue (buckling load) is computed by a series of iterations displaying: iteration count, Rayleigh quotient and the accelerated estimate of the eigenvalue. To reduce the number of iterations required for convergence an initial shift (lower than the expected solution) was utilized.

As a final result, the normalized buckling mode is printed in the same format as the linear solution.

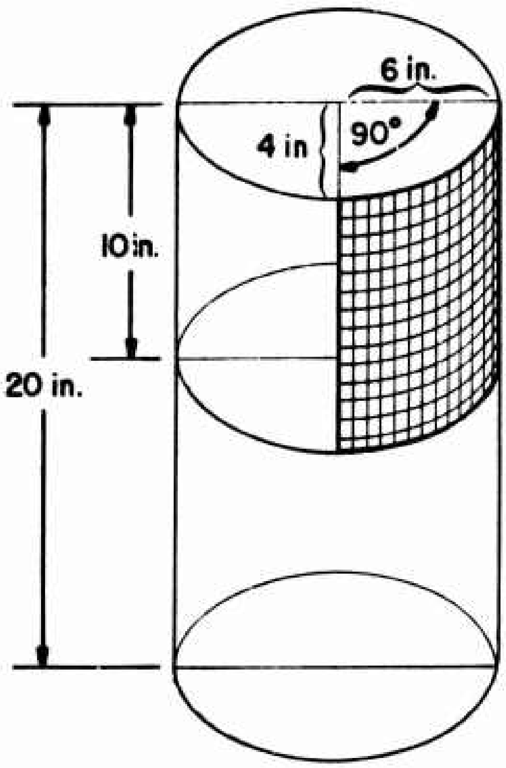


Fig. 7-4 Sample Case 2 – Elliptic Cylinder

SAMPLE CASE 2
INPUT

```
    SAMPLE CASE 2 - ELLIPTIC CYLINDER                         C-1
    6    1    2                                               G-1
 10.         0.         90.        6.          4.             G-2
   15   15                                                    D-1
    0    4    4    4                                          B-1
    0    0    1    0                                          B-2
    1    0 1.                                                 L-1 } System A
0.       0.          200.          0    0    2    1    0      L-2 }
    1    0 1.                                                 L-1 } System B
20.      0.          0.            4    0    0    0    0      L-2 }
         7.0                  1   20                          P-1A
    0    0    0    2    1    1    1                           O-1
5.       10.                                                  O-2
0.                                                            O-3
    2                                                         M-1
 .05        1      + 5  .25                                   M-2B
```

Fig. 7-5 Display of Input Cards for Sample Case 2

SAMPLE CASE 2 – OUTPUT

```
    SAMPLE CASE 2 - ELLIPTIC CYLINDER
BUCKLING ANALYSIS.            2 LOAD PATTERNS.

TYPE OF SURFACE IS ELLPTIC CYL.

SURFACE CONSTANTS =  1.0000000E+01, 0.            , 9.0000000E+01, 6.0000000E+00, 4.0000000E+00,

 BLANK COMMON ARRAY WORKING SPACE=     15000

  FINITE DIFFERENCE MESH.  15 ROWS,   15 COLUMNS.  MESH SPACING. H=     .7143, K=    6.4286

NRM1=  -3, NRM2= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS SET BY IFREE . IFREE = 0, 0, 1, 0,
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =     1

 USER-LOAD FLAG =     0,  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP = -0.          , MAXIMUM LOAD =  1.0000000E+00

      PZ            PY           PX           JZ   JY   JX  ROW  COL
0.            0.           2.0000000E+02     0    0    2    1    0

LOAD B DATA

CARD COUNT =     1

 USER-LOAD FLAG =     0,  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP = -0.          , MAXIMUM LOAD =  1.0000000E+00

      PZ            PY           PX           JZ   JY   JX  ROW  COL
2.0000000E+01 0.            0.                4    0    0    0    0

 ISHIFT    ITERAT    SHIFT
   1         20       7.0000000E+00

IPX=    2 IPY=    1 IPRO=    1 IPRS=    1 IPLOT=    -0

IWALL=    2,  NSTRI=   -0,  NRING=   -0,  IP=   -0,  IN=   -0,  JN=   -0

   AT =  5.0000000E-02  EX1 =  1.0000000E+07 XNU =  2.5000000E-01
    Z = -3.             EY1 =  1.0000000E+07   G =  4.0000000E+06

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CFB2
       CCC(I,1)               CCC(I,2)               CCC(I,3)               CCC(I,4)               CCC(I,5)               CCC(I,6)

                                                                                                                         0.
                                                                            0.                     0.                     0.
                                                     0.                     0.                     0.                     0.
         5.3333333E+05          3.                   0.                     0.                     0.                     0.
         1.3333333E+05          5.3333332E+05        2.0000000E+05          1.1111111E+02          0.                     0.
         0.                     0.                   0.                     2.7777779E+01          1.1111111E+02          4.1666667E+01
         0.                     0.                   0.                     0.                     0.
         0.                     0.                   0.
         0.                     0.
```

Fig. 7-6 Excerpt of Output for Sample Case 2

```
CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=    3.718.  NR OF IO REQUESTS (TAPE2)=    24.      WORDS USED (TAPE2)=   93312.     WORDS TRANSFERRED (TAPE2)=       99456

FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=   12.543.  NR OF IO REQUESTS (TAPE2)=    49.      WORDS USED (TAPE2)=  136448.     WORDS TRANSFERRED (TAPE2)=      219984

ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=   13.72C.  NR OF IO REQUESTS (TAPE2)=    75.      WORDS USED (TAPE2)=  199672.     WORDS TRANSFERRED (TAPE2)=      375424

DETERMINANT OF STIFFNESS MATRIX=  1.2508971E+00*10.**  1948.   NUMBER OF NEGATIVE ROOTS =   0
     225 NODES,        867 EQUATIONS.   MAXIMUM BAND WIDTH =        108
MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=   19.856.  NR OF IO REQUESTS (TAPE2)=   113.      WORDS USED (TAPE2)=  188672.     WORDS TRANSFERRED (TAPE2)=      612480

LINEAR SOLUTION.  PA=   1.00000CE+00,    PB=   0.
ROW 14,  X=      9.286, AXIAL LOAD=      -1.587131E+03,BENDING MOMENT=     0.
```

ROW 8, X= 5.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	2.689605E-04	0.	1.998129E-03	4.252820E-07	0.
2	6.4286	2.849241E-04	-8.563558E-07	1.998154E-03	5.261985E-07	4.631963E-05
3	12.8571	3.294853E-04	-2.331761E-06	1.998290E-03	7.907699E-07	9.335177E-05
4	19.2857	3.942135E-04	-4.391373E-06	1.998501E-03	1.055120E-06	1.067820E-04
5	25.7143	4.682475E-04	-6.499635E-06	1.998838E-03	1.181232E-06	1.173558E-04
6	32.1429	5.445377E-04	-8.130127E-06	1.999274E-03	1,042253E-06	1.181246E-04
7	38.5714	6.152254E-04	-9.098608E-06	1.999769E-03	6.230048E-07	1.122751E-04
8	45.0000	6.776354E-04	-9.249537E-06	2.000279E-03	-7.594890E-09	1.022722E-04
9	51.4286	7.304376E-04	-8.923485E-06	2.000763E-03	-7.462351E-07	8.978668E-05
10	57.8571	7.737269E-04	-7.913813E-06	2.001194E-03	-1.495081E-06	7.587623E-05
11	64.2857	8.078392E-04	-6.661551E-06	2.001557E-03	-2.178851E-06	6.118318E-05
12	70.7143	8.334173E-04	-5.170353E-06	2.001842E-03	-2.747335E-06	4.603357E-05
13	77.1429	8.511955E-04	-3.524311E-06	2.002045E-03	-3.169652E-06	3.078852E-05
14	83.5714	8.610584E-04	-1.783774E-06	2.002166E-03	-3.428580E-06	1.540905E-05
15	90.0000	8.650995E-04	0.	2.002209E-03	-3.515717E-06	0.

ROW 15, X= 10.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	2.702376E-04	0.	-2.168404E-19	0.	0.
2	6.4286	2.863802E-04	-1.193239E-06	-4.336809E-19	-8.775261E-19	4.694730E-05
3	12.8571	3.314243E-04	-3.047317E-06	0.	-7.589415E-19	9.433946E-05
4	19.2857	3.966803E-04	-5.525369E-06	-2.168404E-19	-2.059984E-18	1.076602E-04
5	25.7143	4.715782E-04	-9.042831E-06	-4.336809E-19	-6.776264E-19	1.176134E-04
6	32.1429	5.469569E-04	-1.001647E-05	-2.168404E-19	-2.059984E-18	1.174144E-04
7	38.5714	6.166838E-04	-1.114929E-05	-4.336809E-19	-3.117981E-18	1.105231E-04
8	45.0000	6.775936E-04	-1.146999E-05	0.	-9.486769E-19	9.965832E-05
9	51.4286	7.286563E-04	-1.091293E-05	0.	-2.873136E-18	8.664469E-05
10	57.8571	7.730095E-04	-9.819762E-06	0.	-2.466568E-18	7.258898E-05
11	64.2857	8.023443E-04	-8.286993E-06	0.	-5.583641E-18	5.818963E-05
12	70.7143	8.264799E-04	-6.445439E-06	0.	-2.737618E-19	4.352129E-05
13	77.1429	8.431594E-04	-4.390397E-06	0.	-1.680513E-19	2.895952E-05
14	83.5714	8.529221E-04	-2.228595E-06	0.	-2.744387E-18	1.445878E-05
15	90.0000	8.561405E-04	0.	0.	0.	0.

Fig. 7-6 (Cont.)

ROW 8, X= 5.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.998212E+02	6.577382E-04	0.	2.149599E-03	8.590842E-03	0.
2	6.4286	-1.998225E+02	4.312437E-04	-3.043271E-03	2.167846E-03	8.646134E-03	-2.598594E-05
3	12.8571	-1.998289E+02	-8.084775E-05	-4.409205E-03	2.208737E-03	8.766906E-03	-3.986514E-05
4	19.2857	-1.998461E+02	-5.233422E-04	-3.583550E-03	2.245152E-03	8.869660E-03	-3.399226E-05
5	25.7143	-1.998776E+02	-6.474841E-04	-1.368578E-03	2.259853E-03	8.909859E-03	-8.943906E-06
6	32.1429	-1.999217E+02	-4.636579E-04	9.562848E-04	2.252372E-03	8.895353E-03	2.700356E-05
7	38.5714	-1.999734E+02	-1.512951E-04	2.526829E-03	2.231499E-03	8.855114E-03	6.295476E-05
8	45.0000	-2.000267E+02	1.129888E-04	3.144933E-03	2.206454E-03	8.812321E-03	9.351875E-05
9	51.4286	-2.000773E+02	2.431933E-04	3.034143E-03	2.183062E-03	8.779789E-03	1.056973E-04
10	57.8571	-2.001212E+02	2.388843E-04	2.528136E-03	2.163813E-03	8.758295E-03	1.090993E-04
11	64.2857	-2.001578E+02	1.434309E-04	1.894874E-03	2.149175E-03	8.746370E-03	9.932037E-05
12	70.7143	-2.001853E+02	9.375439E-06	1.291847E-03	2.138732E-03	8.740897E-03	8.173744E-05
13	77.1429	-2.002065E+02	-1.130410E-04	7.319192E-04	2.131342E-03	8.739040E-03	5.753714E-05
14	83.5714	-2.002185E+02	-2.063478E-04	3.639133E-04	2.127945E-03	8.738735E-03	2.959743E-05
15	90.0000	-2.002225E+02	-2.384434E-04	0.	2.126690E-03	8.738766E-03	0.

ROW 15, X= 10.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.998384E+02	8.547888E-04	0.	2.131692E-03	8.478461E-03	0.
2	6.4286	-1.998131E+02	6.397126E-04	-1.355253E-15	2.152011E-03	8.555295E-03	-3.350932E-17
3	12.8571	-1.998274E+02	9.653377E-05	-2.710505E-15	2.200750E-03	8.738523E-03	-3.520405E-19
4	19.2857	-1.998521E+02	-5.257263E-04	-1.034202E-14	2.251942E-03	8.929665E-03	2.256831E-17
5	25.7143	-1.998872E+02	-9.734499E-04	0.	2.284268E-03	9.052571E-03	-4.535455E-17
6	32.1429	-1.999307E+02	-1.114355E-03	0.	2.289800E-03	9.084185E-03	4.301734E-17
7	38.5714	-1.999792E+02	-9.607162E-04	-1.084202E-14	2.271697E-03	9.040995E-03	-1.241799E-16
8	45.0000	-2.000285E+02	-6.139642E-04	-2.168404E-14	2.233140E-03	8.953495E-03	-1.071735E-16
9	51.4286	-2.000761E+02	-1.333252E-04	0.	2.197599E-03	8.849261E-03	-1.285954E-15
10	57.8571	-2.001185E+02	2.275073E-04	0.	2.156681E-03	8.747176E-03	-5.331843E-17
11	64.2857	-2.001544E+02	5.850511E-04	0.	2.119781E-03	8.657993E-03	-8.660470E-17
12	70.7143	-2.001827E+02	8.628829E-04	0.	2.089504E-03	8.536576E-03	-1.100038E-16
13	77.1429	-2.002031E+02	1.057045E-03	-2.718505E-15	2.067256E-03	8.535793E-03	-3.043573E-17
14	83.5714	-2.002154E+02	1.170779E-03	-6.776264E-16	2.053717E-03	8.505196E-03	-5.432631E-17
15	90.0000	-2.002194E+02	1.208198E-03	0.	2.049178E-03	8.495020E-03	0.

COL 1, Y= 0.0000

ROW	X	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.998992E+02	-4.997480E+01	0.	-1.100960E-01	-1.288294E-02	0.
2	.7143	-1.998476E+02	-1.151439E+00	0.	5.773600E-02	2.353558E-02	0.
3	1.4286	-1.998295E+02	2.811940E-01	0.	4.247962E-03	9.406692E-03	0.
4	2.1429	-1.998404E+02	1.431739E-02	0.	1.963713E-03	8.732553E-03	0.
5	2.8571	-1.998356E+02	-1.554457E-03	0.	2.157307E-03	8.770996E-03	0.
6	3.5714	-1.998300E+02	2.431464E-04	0.	2.169673E-03	8.697571E-03	0.
7	4.2857	-1.998253E+02	5.706366E-04	0.	2.157542E-03	8.636772E-03	0.
8	5.0000	-1.998212E+02	6.577382E-04	0.	2.149590E-03	8.590842E-03	0.
9	5.7143	-1.998173E+02	7.234035E-04	0.	2.143891E-03	8.555859E-03	0.
10	6.4286	-1.998149E+02	7.716136E-04	0.	2.139635E-03	8.529356E-03	0.
11	7.1429	-1.998125E+02	8.054311E-04	0.	2.136511E-03	8.509615E-03	0.
12	7.8571	-1.998107E+02	8.285833E-04	0.	2.134292E-03	8.495389E-03	0.
13	8.5714	-1.998094E+02	8.436127E-04	0.	2.132813E-03	8.435801E-03	0.
14	9.2857	-1.998087E+02	8.520579E-04	0.	2.131967E-03	8.480269E-03	0.
15	10.0000	-1.998384E+02	8.547888E-04	0.	2.131692E-03	8.478461E-03	0.

Fig. 7-6 (Cont.)

LINEAR SOLUTION. PA= 0. , PB= 1.000000E+00
ROW 14, X= 9.286, AXIAL LOAD= -3.268303E-01, BENDING MOMENT= 0.

ROW 8, X= 5.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-2.992952E-02	0.	5.524307E-03	-4.524118E-03	0.
2	6.4286	-2.720742E-02	7.229223E-03	5.224999E-03	-4.122480E-03	5.038965E-03
3	12.8571	-1.990579E-02	1.277617E-02	4.404209E-03	-3.045428E-03	8.998497E-03
4	19.2857	-1.000495E-02	1.592829E-02	3.253105E-03	-1.586793E-03	1.121711E-02
5	25.7143	4.636702E-04	1.694235E-02	1.999003E-03	-5.015057E-05	1.168337E-02
6	32.1429	1.005594E-02	1.647316E-02	7.762290E-04	1.347024E-03	1.085400E-02
7	38.5714	1.805302E-02	1.513440E-02	-2.886862E-04	2.496773E-03	9.312187E-03
8	45.0000	2.430125E-02	1.335876E-02	-1.173896E-03	3.378212E-03	7.536899E-03
9	51.4286	2.896633E-02	1.140561E-02	-1.882009E-03	4.020102E-03	5.929335E-03
10	57.3571	3.233464E-02	9.413215E-03	-2.433063E-03	4.469707E-03	4.337423E-03
11	64.2857	3.469734E-02	7.445861E-03	-2.854306E-03	4.774389E-03	3.100927E-03
12	70.7143	3.629751E-02	5.525979E-03	-3.152967E-03	4.973527E-03	2.102086E-03
13	77.1429	3.731893E-02	3.653586E-03	-3.357537E-03	5.096417E-03	1.293127E-03
14	83.5714	3.789434E-02	1.817237E-03	-3.475366E-03	5.162721E-03	6.138846E-04
15	90.0000	3.806534E-02	0.	-3.514572E-03	5.183618E-03	0.

ROW 15, X= 10.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-4.166132E-02	0.	0.	0.	0.
2	6.4286	-3.789938E-02	1.002863E-02	-4.336809E-19	9.237403E-17	6.950795E-03
3	12.8571	-2.780796E-02	1.772437E-02	-4.336809E-19	1.444157E-16	1.241461E-02
4	19.2857	-1.412568E-02	2.209769E-02	-8.673617E-19	5.651404E-17	1.547025E-02
5	25.7143	3.308410E-04	2.350370E-02	-2.168404E-19	-9.673617E-19	1.609974E-02
6	32.1429	1.355223E-02	2.284826E-02	-9.486769E-20	-4.250073E-17	1.490059E-02
7	38.5714	2.453679E-02	2.098586E-02	1.084202E-19	-1.682682E-16	1.271737E-02
8	45.0000	3.307499E-02	1.851769E-02	0.	-7.979728E-17	1.021613E-02
9	51.4286	3.940324E-02	1.580475E-02	0.	-1.821460E-16	7.825763E-03
10	57.3571	4.393399E-02	1.303949E-02	0.	-7.235939E-17	5.754715E-03
11	64.2857	4.708114E-02	1.031114E-02	2.168404E-19	-2.688821E-16	4.061195E-03
12	70.7143	4.919225E-02	7.650566E-03	8.673617E-19	-2.654107E-16	2.717299E-03
13	77.1429	5.052733E-02	5.057364E-03	0.	-2.194425E-16	1.652483E-03
14	83.5714	5.126183E-02	2.515174E-03	4.336809E-19	-1.912533E-16	7.732399E-04
15	90.0000	5.149535E-02	0.	4.336809E-19	0.	0.

COL 1, Y= 0.0000

ROW	X	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	9.131114E-03	0.	0.
2	.7143	-4.628975E-03	0.	8.035650E-03	-6.662578E-03	0.
3	1.4286	-9.517969E-03	0.	7.846179E-03	-6.682023E-03	0.
4	2.1429	-1.417472E-02	0.	7.554185E-03	-6.346750E-03	0.
5	2.8571	-1.858476E-02	0.	7.167139E-03	-5.972444E-03	0.
6	3.5714	-2.270673E-02	0.	6.693852E-03	-5.540867E-03	0.
7	4.2857	-2.650029E-02	0.	6.143315E-03	-5.059913E-03	0.
8	5.0000	-2.992952E-02	0.	5.524307E-03	-4.524118E-03	0.
9	5.7143	-3.296331E-02	0.	4.849558E-03	-3.951796E-03	0.
10	6.4286	-3.557494E-02	0.	4.115729E-03	-3.345053E-03	0.
11	7.1429	-3.774197E-02	0.	3.343424E-03	-2.709867E-03	0.
12	7.8571	-3.944018E-02	0.	2.537198E-03	-2.052061E-03	0.
13	8.5714	-4.067348E-02	0.	1.705565E-03	-1.377378E-03	0.
14	9.2857	-4.141380E-02	0.	8.570135E-04	-6.914355E-04	0.
15	10.0000	-4.166132E-02	0.	0.	0.	0.

Fig. 7-6 (Cont.)

ROW 8, X= 5.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-4.421756E+02	5.424418E+01	0.	-2.932653E-01	-9.701255E-01	0.
2	6.4286	-4.168152E+02	5.680549E+01	3.811199E+01	-2.657935E-01	-7.690750E-01	-6.146474E-02
3	12.8571	-3.473079E+02	6.424519E+01	7.228172E+01	-1.823739E-01	-5.125734E-01	-1.097409E-01
4	19.2857	-2.499143E+02	7.577205E+01	9.923958E+01	-7.935103E-02	-2.046086E-01	-1.364898E-01
5	25.7143	-1.429646E+02	9.007558E+01	1.170841E+02	1.460435E-02	6.199600E-02	-1.412453E-01
6	32.1429	-4.056301E+01	1.056174E+02	1.255741E+02	8.461960E-02	2.430699E-01	-1.236293E-01
7	38.5714	4.946227E+01	1.210112E+02	1.257769E+02	1.285641E-01	3.373999E-01	-1.031038E-01
8	45.0000	1.243707E+02	1.352949E+02	1.193896E+02	1.511321E-01	3.649617E-01	-8.535236E-02
9	51.4286	1.844398E+02	1.477609E+02	1.391593E+02	1.589070E-01	3.499659E-01	-5.419965E-02
10	57.8571	2.312492E+02	1.592662E+02	9.356947E+01	1.573329E-01	3.129165E-01	-4.577541E-02
11	64.2857	2.665391E+02	1.667130E+02	7.675197E+01	1.524166E-01	2.686447E-01	-3.118343E-02
12	70.7143	2.924695E+02	1.731492E+02	5.851291E+01	1.457598E-01	2.269317E-01	-2.010515E-02
13	77.1429	3.099384E+02	1.776605E+02	3.940703E+01	1.398561E-01	1.938155E-01	-1.181775E-02
14	83.5714	3.202489E+02	1.803295E+02	1.981276E+01	1.358997E-01	1.727774E-01	-5.430972E-03
15	90.0000	3.233579E+02	1.812125E+02	0.	1.345163E-01	1.655856E-01	0.

ROW 15, X= 10.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-5.877073E+02	5.471735E+01	0.	-4.009419E-01	-1.199536E+00	0.
2	6.4286	-5.545363E+02	5.719546E+01	0.	-3.572813E-01	-1.060705E+00	6.253046E-15
3	12.8571	-4.633367E+02	6.442021E+01	2.236757E-11	-2.448859E-01	-7.071072E-01	1.432952E-14
4	19.2857	-3.350761E+02	7.568169E+01	1.539327E-13	-1.066563E-01	-2.803982E-01	1.030563E-14
5	25.7143	-1.933585E+02	8.975434E+01	2.227703E-11	2.154921E-02	9.096103E-02	1.597392E-14
6	32.1429	-5.635300E+01	1.051537E+02	5.553696E-11	1.160693E-01	3.438199E-01	1.167592E-14
7	38.5714	6.379759E+01	1.205229E+02	2.227029E-11	1.745594E-01	4.746441E-01	1.955934E-15
8	45.0000	1.646381E+02	1.348295E+02	0.	2.032832E-01	5.105249E-01	5.089495E-15
9	51.4286	2.457738E+02	1.474589E+02	0.	2.113697E-01	4.854836E-01	-4.938355E-15
10	57.8571	3.091551E+02	1.531152E+02	1.134376E-11	2.072129E-01	4.233511E-01	-9.075871E-15
11	64.2857	3.572193E+02	1.667135E+02	5.551115E-12	1.972943E-01	3.620867E-01	-1.305239E-15
12	70.7143	3.922205E+02	1.732826E+02	5.551115E-12	1.859620E-01	2.994598E-01	-2.060649E-15
13	77.1429	4.159554E+02	1.778958E+02	2.775558E-12	1.762253E-01	2.498290E-01	-1.973137E-14
14	83.5714	4.296967E+02	1.806283E+02	-0.	1.697803E-01	2.193241E-01	-6.949734E-15
15	90.0000	4.341949E+02	1.815329E+02	0.	1.675374E-01	2.075574E-01	0.

COL 1, Y= 0.0000

ROW	X	NX	NY	NXY	MX	MY	MXY
1	0.0000	-4.046266E+01	-1.011567E+01	0.	2.016966E+00	5.072051E-01	0.
2	.7143	-8.011897E+01	6.230691E+01	0.	1.072801E-02	-1.374365E-01	0.
3	1.4286	-1.560644E+02	5.396609E+01	0.	-1.220938E-01	-2.987121E-01	0.
4	2.1429	-2.258349E+02	5.355320E+01	0.	-1.571523E-01	-4.271273E-01	0.
5	2.8571	-2.892407E+02	5.376824E+01	0.	-1.965275E-01	-5.508948E-01	0.
6	3.5714	-3.463901E+02	5.394609E+01	0.	-2.337964E-01	-6.668941E-01	0.
7	4.2857	-3.973492E+02	5.410336E+01	0.	-2.677852E-01	-7.736519E-01	0.
8	5.0000	-4.421756E+02	5.424413E+01	0.	-2.932658E-01	-8.701255E-01	0.
9	5.7143	-4.809207E+02	5.436784E+01	0.	-3.250497E-01	-9.554267E-01	0.
10	6.4286	-5.136293E+02	5.447356E+01	0.	-3.479741E-01	-1.023303E+00	0.
11	7.1429	-5.403347E+02	5.456178E+01	0.	-3.669054E-01	-1.089654E+00	0.
12	7.8571	-5.610806E+02	5.462904E+01	0.	-3.817369E-01	-1.137474E+00	0.
13	8.5714	-5.758793E+02	5.467803E+01	0.	-3.923876E-01	-1.171494E+00	0.
14	9.2857	-5.847511E+02	5.470751E+01	0.	-3.988805E-01	-1.192650E+00	0.
15	10.0000	-5.877373E+02	5.471735E+01	0.	-4.009419E-01	-1.199586E+00	0.

Fig. 7-6 (Cont.)

ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS= 34.030. NR OF IO REQUESTS (TAPE2)= 289. WORDS USED (TAPE2)= 190728. WORDS TRANSFERRED (TAPE2)= 1471872

DETERMINANT OF STIFFNESS MATRIX= 7.3297253E+03*10.** 1548. NUMBER OF NEGATIVE ROOTS = 0
225 NODES. 667 EQUATIONS. MAXIMUM BAND WIDTH = 108
MATRIX DECOMPOSITION COMPLETED.
CP SECONDS= 39.379. NR OF IO REQUESTS (TAPE2)= 319. WORDS USED (TAPE2)= 190728. WORDS TRANSFERRED (TAPE2)= 1789928

EIGENVALUE SHIFT= 7.0000000E+00, NUMBER OF NEGATIVE ROOTS= 0

ITERATION	EIGENVALUE (RAYLEIGH QUOTIENT)	EIGENVALUE (ACCELERATED ESTIMATE)
0	8.6126638E+02	7.0000000E+00
1	1.9972165E+01	6.7311899E+00
2	1.6392447E+01	1.7213825E+01
3	1.5382376E+01	1.4290537E+01
4	1.4459944E+01	1.3394095E+01
5	1.4158495E+01	1.3978604E+01
6	1.4013320E+01	1.3877623E+01
7	1.3941125E+01	1.3969707E+01
8	1.3921184E+01	1.3951722E+01
9	1.3374267E+01	1.3818638E+01
10	1.3851900E+01	1.3741974E+01
11	1.3830716E+01	1.3450366E+01
12	1.3809631E+01	9.3164597E+00
13	1.3733592E+01	2.4099523E+01
14	1.3767511E+01	1.0563533E+01
15	1.3746995E+01	1.2613934E+01

ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS= 59.326. NR OF IO REQUESTS (TAPE2)= 765. WORDS USED (TAPE2)= 238912. WORDS TRANSFERRED (TAPE2)= 4452488

DETERMINANT OF STIFFNESS MATRIX= 2.1152429E+08*10.** 1539. NUMBER OF NEGATIVE ROOTS = 0
225 NODES. 667 EQUATIONS. MAXIMUM BAND WIDTH = 108
MATRIX DECOMPOSITION COMPLETED.
CP SECONDS= 64.724. NR OF IO REQUESTS (TAPE2)= 803. WORDS USED (TAPE2)= 238912. WORDS TRANSFERRED (TAPE2)= 4689536

EIGENVALUE SHIFT= 1.1722826E+01, NUMBER OF NEGATIVE ROOTS= 0

ITERATION	EIGENVALUE (RAYLEIGH QUOTIENT)	EIGENVALUE (ACCELERATED ESTIMATE)
0	1.3708087E+01	1.3790874E+01
1	1.3647719E+01	1.3916743E+01
2	1.3592539E+01	1.3034597E+01
3	1.3542597E+01	1.3068274E+01
4	1.3499080E+01	1.3203851E+01
5	1.3463149E+01	1.3292971E+01
6	1.3435159E+01	1.3336486E+01
7	1.3414505E+01	1.3356372E+01
8	1.3399974E+01	1.3365488E+01
9	1.3390170E+01	1.3369837E+01
10	1.3383812E+01	1.3372080E+01
11	1.3379360E+01	1.3373371E+01
12	1.3377534E+01	1.3374236E+01
13	1.3376275E+01	1.3374790E+01
14	1.3375699E+01	1.3375208E+01

THE BUCKLING LOAD BASED ON LINEAR BIFURCATION THEORY IS 1.337521E+01 TIMES THE STARTING LOAD.

Fig. 7-6 (Cont.)

ROW 8, X= 5.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	4.321592E-02	0.	-7.080786E-03	9.015740E-02	0.
2	6.4286	-3.754991E-02	-1.115650E-03	-2.236417E-03	4.639423E-02	-1.170621E-01
3	12.8571	-1.151036E-01	1.572259E-02	6.764254E-03	-9.554679E-02	5.918444E-02
4	19.2857	3.758341E-02	2.503972E-02	8.632917E-03	-1.830846E-01	2.885529E-01
5	25.7143	2.734324E-01	2.453382E-03	-5.052837E-04	-1.416805E-02	1.415650E-01
6	32.1429	2.423860E-01	-3.162015E-02	-1.122562E-02	2.972331E-01	-2.373503E-01
7	38.5714	-7.146437E-02	-4.309597E-02	-1.237037E-02	3.588460E-01	-5.060297E-01
8	45.0000	-3.716025E-01	-2.515005E-02	-3.318245E-03	8.327463E-02	-3.059057E-01
9	51.4286	-4.400726E-01	5.174501E-03	8.071886E-03	-3.166258E-01	8.900656E-02
10	57.8571	-2.905994E-01	2.964353E-02	1.425761E-02	-5.532399E-01	3.755365E-01
11	64.2857	-5.727170E-02	4.037233E-02	1.269735E-02	-4.947335E-01	4.364188E-01
12	70.7143	1.434421E-01	3.846276E-02	5.301445E-03	-1.915896E-01	3.340408E-01
13	77.1429	2.601323E-01	2.855328E-02	-4.008313E-03	2.426257E-01	1.870567E-01
14	83.5714	3.205262E-01	1.489077E-02	-1.136551E-02	5.203358E-01	7.327494E-02
15	90.0000	3.347771E-01	0.	-1.412590E-02	6.406703E-01	0.

ROW 15, X= 10.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-7.025973E-02	0.	-1.734723E-18	0.	0.
2	6.4286	-1.665225E-03	9.216305E-03	0.	-1.561251E-17	1.419264E-01
3	12.8571	1.230975E-01	-3.581244E-03	3.469447E-18	-2.567391E-16	6.812803E-02
4	19.2857	9.638350E-02	-2.527480E-02	1.084202E-18	-2.914335E-16	-2.208665E-01
5	25.7143	-1.649328E-01	-2.226558E-02	-3.469447E-18	4.302114E-16	-3.700270E-01
6	32.1429	-3.947905E-01	1.261938E-02	0.	3.326673E-16	-6.977605E-02
7	38.5714	-2.776130E-01	4.962226E-02	0.	1.214306E-15	4.791106E-01
8	45.0000	1.617595E-01	5.695323E-02	0.	-7.494005E-16	7.810281E-01
9	51.4286	6.167306E-01	2.857151E-02	6.938894E-18	-2.053913E-15	5.678401E-01
10	57.8571	7.963441E-01	-1.861337E-02	1.387779E-17	-8.326673E-16	-3.212414E-02
11	64.2857	6.043164E-01	-6.123945E-02	6.933394E-19	-4.510281E-16	-6.808720E-01
12	70.7143	1.356619E-01	-8.240505E-02	2.602085E-18	-6.106227E-16	-1.069535E+00
13	77.1429	-4.136172E-01	-7.568635E-02	-6.938894E-18	9.506285E-16	-1.046841E+00
14	83.5714	-8.404526E-01	-4.474047E-02	0.	1.353084E-15	-6.374716E-01
15	90.0000	-1.000000E+00	0.	0.	0.	0.

COL 1, Y= 0.0000

ROW	X	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	5.892568E-03	0.	0.
2	.7143	-7.975959E-02	0.	6.449137E-03	-1.131766E-01	0.
3	1.4286	-1.516808E-01	0.	6.028812E-03	-8.860778E-02	0.
4	2.1429	-2.063421E-01	0.	3.858973E-03	-2.224949E-02	0.
5	2.8571	-1.934658E-01	0.	1.583649E-04	5.347089E-02	0.
6	3.5714	-1.299551E-01	0.	-3.705292E-03	1.066974E-01	0.
7	4.2857	-4.124097E-02	0.	-5.357132E-03	1.212197E-01	0.
8	5.0000	4.321592E-02	0.	-7.080786E-03	9.015740E-02	0.
9	5.7143	9.918389E-02	0.	-5.776354E-03	5.056022E-02	0.
10	6.4286	1.154448E-01	0.	-3.148426E-03	-3.979205E-03	0.
11	7.1429	9.349930E-02	0.	-2.510307E-04	-4.902553E-02	0.
12	7.8571	4.545935E-02	0.	1.874169E-03	-7.265212E-02	0.
13	8.5714	-1.025945E-02	0.	2.561735E-03	-6.950052E-02	0.
14	9.2857	-5.357812E-02	0.	1.751844E-03	-4.200020E-02	0.
15	10.0000	-7.028973E-02	0.	-1.734723E-18	0.	0.

CP SECONDS= 70.798. NR OF IO REQUESTS (TAPE2)= 1126. WORDS USED (TAPE2)= 230912. WORDS TRANSFERRED (TAPE2)= 6769024

Fig. 7-6 (Cont.)

7.3 SAMPLE CASE 3 – CYLINDER WITH RECTANGULAR CUTOUT

This case demonstrates calculation of the buckling load, according to bifurcation theory, for a cylinder with rectangular cutout (Fig. 7-7) and subjected to a uniform line load applied on boundary line 1. Because of symmetry, only one eighth of the shell needs to be analyzed. The input cards associated with this case are illustrated in Fig. 7-8. Portions of the output are presented in Fig. 7-9. The output is the same as in Sample Case 2.

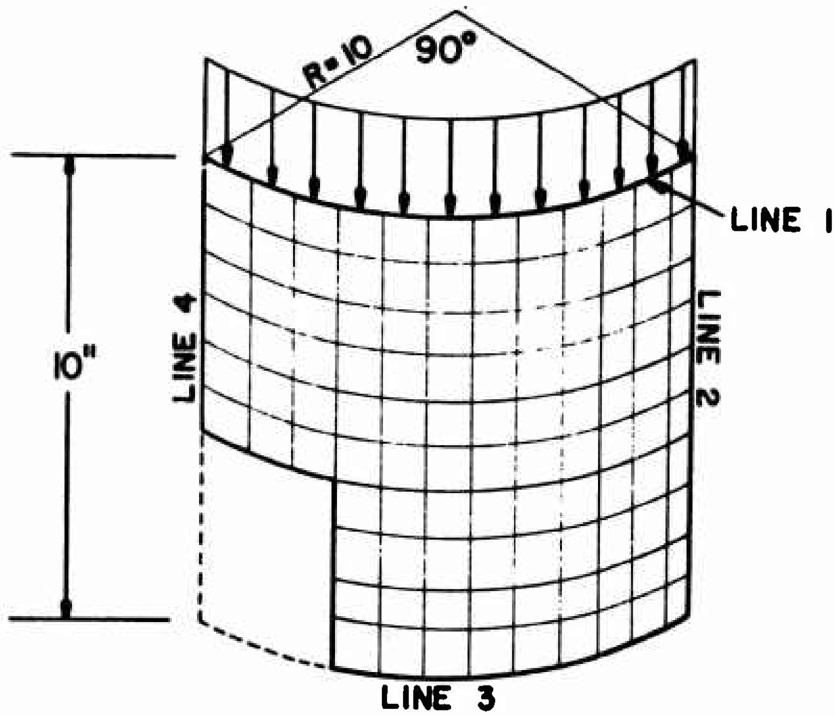


Fig. 7-7 Sample Case 3 – Cylinder With Rectangular Cutout

SAMPLE CASE 3
INPUT

```
    SAMPLE CASE 3 - CYLINDER WITH RECTANGULAR CUTOUT              C-1
    1      1      1                                               G-1
10.0       90.0        10.0                                       G-2
   11     13      7     11     4                                  D-1
    1      4      4      4                                        B-1
    1      0 1.                                                   L-1
0.         0.          1.          0      0      2      1     0   L-2
0.0        0.0                0      0     20                     P-1A
    0      0      0      1      2      1      1                   O-1
10.                                                               O-2
22.5       90.                                                    O-3
    2                                                             M-1
0.1        10000000. 0.3         0.                               M-2B
```

Fig. 7-8 Display of Input Cards for Sample Case 3

```
    SAMPLE CASE 3 - CYLINDER WITH RECTANGULAR CUTOUT
BUCKLING ANALYSIS.            1 LOAD PATTERNS.

TYPE OF SURFACE IS CYLINDER

SURFACE CONSTANTS =  1.0000000E+01, 9.0000000E+01, 1.0000000E+01,

 BLANK COMMON ARRAY WORKING SPACE=      15000

  FINITE DIFFERENCE MESH.  11 ROWS,   13 COLUMNS.  MESH SPACING, H=   1.0000, K=   7.5000

NRW1=   7, NRW2= 11, NCL1=   4
BOUNDARY CONDITION AT LINE 1 IS SIMPLESUPRT.
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =      1

 USER-LOAD FLAG =      0.  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP = -0.          ,  MAXIMUM LOAD =  1.0000000E+00

       PZ            PY              PX          JZ    JY    JX   ROW   COL
 0.            0.            1.0000000E+00     0     0     2     1     0

 ISHIFT   ITERAT   SHIFT
   0        20       3.

IPX=     1 IPY=     2 IPRD=     1 IPRS=     1 IPLOT=    -0

INALL=     2.  NSTRI=   -0.  NRING=   -0.  IP=   -0.  IM=   -0.  JM=   -0

  AT =  1.0000000E-01  EX1 =  1.0000000E+07 XNU =  3.0000000E-01
   Z =  0.             EY1 =  1.0000000E+07   G =  3.8461538E+06

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CFB2
     CCC(I,1)            CCC(I,2)            CCC(I,3)            CCC(I,4)            CCC(I,5)            CCC(I,6)

        1.098901E+06        0.                  0.                  0.                  0.                  0.
        3.296703E+05        1.098901E+06        0.                  0.                  0.                  0.
        0.                  0.                  3.846154E+05        0.                  0.                  0.
        0.                  0.                  0.                  9.157509E+02        0.                  0.
        0.                  0.                  0.                  2.747253E+02        9.157509E+02        0.
        0.                  0.                  0.                  0.                  0.                  3.205128E+02
```

Fig. 7-9 Excerpt of Output for Sample Case 3

```
CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=    1.753.  NR OF IO REQUESTS (TAPE2)=    12.      WORDS USED (TAPE2)=   33872.     WORDS TRANSFERRED (TAPE2)=      37968

FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=    3.464.  NR OF IO REQUESTS (TAPE2)=    26.      WORDS USED (TAPE2)=   59536.     WORDS TRANSFERRED (TAPE2)=      93392

ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=    4.058.  NR OF IO REQUESTS (TAPE2)=    34.      WORDS USED (TAPE2)=   79552.     WORDS TRANSFERRED (TAPE2)=     139072


DETERMINANT OF STIFFNESS MATRIX=   5.2394592E+08*10.**    950.   NUMBER OF NEGATIVE ROOTS =   0
     143 NODES.          585 EQUATIONS.    MAXIMUM BAND WIDTH =          84
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=    6.352.  NR OF IO REQUESTS (TAPE2)=    42.      WORDS USED (TAPE2)=   79552.     WORDS TRANSFERRED (TAPE2)=     181152


LINEAR SOLUTION.  PA=   1.000000E+00.    PB=   0.
ROW 10.  X=      9.000. AXIAL LOAD=       -1.570796E+01.BENDING MOMENT=      0.
```

ROW 11. X= 10.0000

COL	Y	W	V	U	BETAX	BETAY
4	22.5000	2.191537E-04	9.327852E-05	0.	0.	-1.435874E-06
5	30.0000	2.256904E-04	6.589299E-05	-2.541099E-21	0.	-1.118126E-05
6	37.5000	2.071320E-04	3.829360E-05	-1.694066E-21	0.	-2.957141E-05
7	45.0000	1.582979E-04	1.466896E-05	-1.588187E-22	0.	-4.449649E-05
8	52.5000	9.448079E-05	-1.728766E-06	-4.235165E-22	0.	-4.650813E-05
9	60.0000	3.608728E-05	-1.007722E-05	-2.117582E-22	0.	-3.678242E-05
10	67.5000	-4.453557E-06	-1.185187E-05	-8.470329E-22	0.	-2.252948E-05
11	75.0000	-2.599757E-05	-9.464011E-06	-8.470329E-22	0.	-1.034077E-05
12	82.5000	-3.400317E-05	-5.070326E-06	-8.470329E-22	0.	-3.220366E-06
13	90.0000	-3.577682E-05	0.	-8.470329E-22	0.	0.

COL 4. Y= 22.5000

ROW	X	W	V	U	BETAX	BETAY
1	0.0000	0.	6.776264E-21	3.422462E-05	6.088286E-06	-6.776264E-22
2	1.0000	6.086286E-06	1.082080E-05	3.309907E-05	5.751987E-06	1.459121E-05
3	2.0000	1.150397E-05	2.172102E-05	3.174158E-05	6.602992E-06	2.922553E-05
4	3.0000	1.929207E-05	3.277000E-05	3.013640E-05	1.004024E-05	4.395097E-05
5	4.0000	3.158445E-05	4.398627E-05	2.830373E-05	1.563413E-05	5.858060E-05
6	5.0000	5.056032E-05	5.546049E-05	2.620007E-05	2.593361E-05	7.296231E-05
7	6.0000	8.345108E-05	6.720737E-05	2.327166E-05	4.023159E-05	8.382868E-05
8	7.0000	1.310235E-04	7.801989E-05	1.836929E-05	4.829352E-05	7.592821E-05
9	8.0000	1.800387E-04	8.638734E-05	1.219583E-05	3.898843E-05	1.749224E-05
10	9.0000	2.090034E-04	9.154636E-05	6.087656E-06	1.955751E-05	8.093525E-06
11	10.0000	2.191537E-04	9.327852E-05	0.	0.	-1.435874E-06

Fig. 7-9 (Cont.)

COL 13. Y= 90.0000

ROW	X	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	1.224406E-05	-3.321014E-06	0.
2	1.0000	-3.321014E-06	0.	1.125286E-05	-4.346906E-06	0.
3	2.0000	-5.693913E-06	0.	1.017166E-05	-5.500147E-06	0.
4	3.0000	-1.432211E-05	0.	9.832026E-06	-5.428664E-06	0.
5	4.0000	-1.955114E-05	0.	7.841303E-06	-4.933275E-06	0.
6	5.0000	-2.418866E-05	0.	6.609462E-06	-4.334117E-06	0.
7	6.0000	-2.821937E-05	0.	5.334402E-06	-3.604942E-06	0.
8	7.0000	-3.139454E-05	0.	4.031452E-06	-2.807648E-06	0.
9	8.0000	-3.383467E-05	0.	2.699909E-06	-1.905403E-06	0.
10	9.0000	-3.520935E-05	0.	1.354950E-06	-9.710756E-07	0.
11	10.0000	-3.577682E-05	0.	-8.470329E-22	0.	0.

ROW 11. X= 10.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
4	22.5000	-6.084743E+00	2.642859E-02	2.085004E-15	1.926125E-02	7.796681E-03	0.
5	30.0000	-2.732422E+00	-2.917530E-02	1.563753E-15	1.129211E-02	1.377754E-02	0.
6	37.5000	-1.115260E+00	-1.165197E-02	6.255013E-14	7.989659E-03	1.542206E-02	5.212510E-18
7	45.0000	-4.478344E-01	-1.008373E-02	5.570971E-15	4.976670E-03	7.471040E-03	4.343759E-19
8	52.5000	-2.686261E-01	1.913256E-02	-1.068565E-14	8.338760E-04	-3.166574E-03	-8.687517E-19
9	60.0000	-5.262490E-01	4.371621E-02	-1.042502E-14	-2.362190E-03	-9.675086E-03	-5.646886E-17
10	67.5000	-7.810146E-01	1.601209E-02	-2.085004E-14	-2.994043E-03	-1.007919E-02	-1.737503E-18
11	75.0000	-1.124704E+00	1.554134E-02	-1.563753E-14	-2.821916E-03	-7.184811E-03	-1.303128E-18
12	82.5000	-1.267648E+00	-2.032475E-02	-7.818766E-15	-1.574953E-03	-3.189982E-03	-6.515638E-19
13	90.0000	-1.387358E+00	-7.349139E-03	0.	-1.498560E-03	-1.842556E-03	0.

COL 4. Y= 22.5000

ROW	X	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.124400E+00	-3.373201E-01	-2.550511E-01	0.	0.	-9.701305E-03
2	1.0000	-1.238329E+00	-3.478447E-02	-2.457308E-01	1.528530E-03	3.237884E-03	-9.715379E-03
3	2.0000	-1.456211E+00	2.907923E-03	-2.491053E-01	-1.328547E-04	6.141615E-03	-9.762908E-03
4	3.0000	-1.727728E+00	-1.482375E-02	-2.219703E-01	-5.717120E-04	1.060619E-02	-9.764808E-03
5	4.0000	-1.963596E+00	-7.176036E-02	-2.800569E-01	-4.291808E-04	1.713466E-02	-9.664275E-03
6	5.0000	-2.265348E+00	-2.890080E-04	-3.337981E-01	-4.109952E-03	2.495427E-02	-8.462565E-03
7	6.0000	-3.649343E+00	-1.946323E-01	-4.152515E-01	3.287897E-04	4.187507E-02	-1.615203E-03
8	7.0000	-5.355340E+00	-4.727931E-01	-3.107325E-01	1.558497E-02	5.629224E-02	2.096172E-02
9	8.0000	-6.114359E+00	6.547408E-02	-4.698588E-03	1.965750E-02	9.821072E-03	2.184981E-02
10	9.0000	-6.007611E+00	-4.990744E-02	-1.684493E-02	1.996380E-02	1.430067E-02	5.956225E-03
11	10.0000	-6.084743E+00	2.642859E-02	2.085004E-15	1.926125E-02	7.796681E-03	0.

COL 13. Y= 90.0000

ROW	X	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.032268E+00	-3.096303E-01	0.	0.	0.	0.
2	1.0000	-1.054489E+00	-3.953228E-02	0.	1.857810E-03	4.977392E-04	0.
3	2.0000	-1.118750E+00	2.072399E-03	0.	1.873962E-04	-8.727961E-05	0.
4	3.0000	-1.162271E+00	-7.241597E-03	0.	-4.564159E-04	-4.123261E-04	0.
5	4.0000	-1.224900E+00	-1.120555E-02	0.	-6.952999E-04	-6.745669E-04	0.
6	5.0000	-1.247575E+00	-1.846833E-02	0.	-7.632508E-04	-8.586129E-04	0.
7	6.0000	-1.311175E+00	-1.063697E-02	0.	-1.076988E-03	-1.224548E-03	0.
8	7.0000	-1.304941E+00	-2.341921E-02	0.	-1.007161E-03	-1.293204E-03	0.
9	8.0000	-1.357677E+00	-8.352454E-03	0.	-1.386831E-03	-1.674293E-03	0.
10	9.0000	-1.332691E+00	-2.648306E-02	0.	-1.137882E-03	-1.550714E-03	0.
11	10.0000	-1.387358E+00	-7.349139E-03	0.	-1.498560E-03	-1.842556E-03	0.

Fig. 7-9 (Cont.)

EIGENVALUE SHIFT= 0. . NUMBER OF NEGATIVE ROOTS= 0

ITERATION	EIGENVALUE (RAYLEIGH QUOTIENT)	EIGENVALUE (ACCELERATED ESTIMATE)
0	-1.7245027E+03	-7.2759576E-12
1	4.1760507E+02	-7.6912570E+02
2	4.0267102E+02	4.0277441E+02
3	4.1341911E+02	4.0777241E+02
4	4.1537256E+02	4.2207002E+02
5	4.1724309E+02	4.1912996E+02
6	4.1821883E+02	4.1902465E+02
7	4.1865240E+02	4.1899722E+02
8	4.1926344E+02	4.1899394E+02
9	4.1992741E+02	4.1899326E+02
10	4.1995423E+02	4.1899317E+02

THE BUCKLING LOAD BASED ON LINEAR BIFURCATION THEORY IS 4.189932E+02 TIMES THE STARTING LOAD.

ROW 11, X= 10.0000

COL	Y	W	V	U	BETAX	BETAY
4	22.5000	6.764520E-01	8.277289E-02	0.	0.	-2.744063E-01
5	30.0000	3.030565E-01	2.026352E-02	1.301043E-18	0.	-2.305742E-01
6	37.5000	7.811520E-02	-4.100108E-03	6.437450E-20	0.	-1.168595E-01
7	45.0000	-3.970516E-03	-8.387640E-03	-4.336809E-19	0.	-3.750342E-02
8	52.5000	-2.747384E-02	-6.540419E-03	0.	0.	-6.566717E-03
9	60.0000	-2.287442E-02	-3.650200E-03	0.	0.	2.709279E-03
10	67.5000	-1.632559E-02	-1.106917E-03	0.	0.	6.456207E-03
11	75.0000	-6.261398E-03	3.783914E-04	0.	0.	7.325069E-03
12	82.5000	2.939467E-03	6.208633E-04	-5.421011E-20	0.	4.960143E-03
13	90.0000	6.086267E-03	0.	0.	0.	0.

COL 4, Y= 22.5000

ROW	X	W	V	U	BETAX	BETAY
1	0.0000	0.	5.204170E-18	6.185157E-03	1.731803E-03	-5.204170E-19
2	1.0000	1.731803E-03	4.028750E-03	6.149299E-03	1.812116E-03	6.402495E-03
3	2.0000	3.624232E-03	8.110803E-03	6.066200E-03	1.745609E-03	1.295589E-02
4	3.0000	5.223021E-03	1.226706E-02	5.940010E-03	8.584534E-04	1.990746E-02
5	4.0000	5.341139E-03	1.668293E-02	5.745123E-03	-5.782185E-04	2.653754E-02
6	5.0000	4.066584E-03	2.137547E-02	5.595046E-03	3.396373E-03	3.972619E-02
7	6.0000	1.213388E-02	2.730932E-02	5.720913E-03	5.206647E-02	5.756222E-02
8	7.0000	1.097995E-01	3.973297E-02	6.475506E-03	1.703101E-01	6.843677E-02
9	8.0000	3.527540E-01	5.876892E-02	6.438194E-03	2.364165E-01	-1.061297E-01
10	9.0000	5.826326E-01	7.595056E-02	4.027364E-03	1.612490E-01	-2.205566E-01
11	10.0000	6.764520E-01	8.277289E-02	0.	0.	-2.744063E-01

COL 13, Y= 90.0000

ROW	X	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	3.144557E-04	2.645010E-03	0.
2	1.0000	2.645010E-03	0.	3.161101E-04	2.475282E-03	0.
3	2.0000	4.950405E-03	0.	3.134126E-04	2.086616E-03	0.
4	3.0000	6.618243E-03	0.	3.123716E-04	1.494190E-03	0.
5	4.0000	7.938784E-03	0.	3.080812E-04	8.222206E-04	0.
6	5.0000	8.462694E-03	0.	2.903097E-04	2.295208E-04	0.
7	6.0000	8.397026E-03	0.	2.699339E-04	-2.743994E-04	0.
8	7.0000	7.913885E-03	0.	2.188173E-04	-4.680657E-04	0.
9	8.0000	7.461695E-03	0.	1.631956E-04	-4.974154E-04	0.
10	9.0000	6.919355E-03	0.	8.289955E-05	-2.877137E-04	0.
11	10.0000	6.085267E-03	0.	0.	0.	0.

CP SECONDS= 11.412. NR OF IO REQUESTS (TAPE2)= 189. WORDS USED (TAPE2)= 101616. WORDS TRANSFERRED (TAPE2)= 1002284

Fig. 7-9 (Cont.)

7.4 SAMPLE CASE 4 – TOROIDAL SHELL

The critical axial load according to bifurcation theory is to be determined for a toroidal shell segment simply supported at circular edges and stiffened with internal stringers. Because this is a shell of revolution with axially symmetrical loading, it could be analyzed with a one dimensional computer program. Such cases are included here because they provide an opportunity to check the program. The geometry of the shell is shown in Fig. 7-10.

It is assumed here that the critical axial load is obtained for a mode with 36 circumferential waves. Therefore, a shell covering one half the length and 1/144 of the circumference can be analyzed. That is, the shell segment is expected to buckle with one quarter of a sine wave in the circumferential direction. Thus, on the sides formed by the generators (2 and 4), symmetry will be used for both in the prebuckling analysis and symmetry for one and antisymmetry for the other in the buckling analysis. The input cards associated with this case are displayed in Fig. 7-11. Portions of the output are presented in Fig. 7-12.

For zero initial eigenvalue shift, the buckling load obtained was -1780 lb/in. (i.e. tension). To obtain a compressive buckling load, a run was performed with an initial eigenvalue shift of +1780, but the results were again -1780 lb/in. This indicates that the compressive buckling load must be greater than 5340. Therefore, an initial eigenvalue shift of 5340 was tried leading to a bifurcation load of 6390 lb/in.

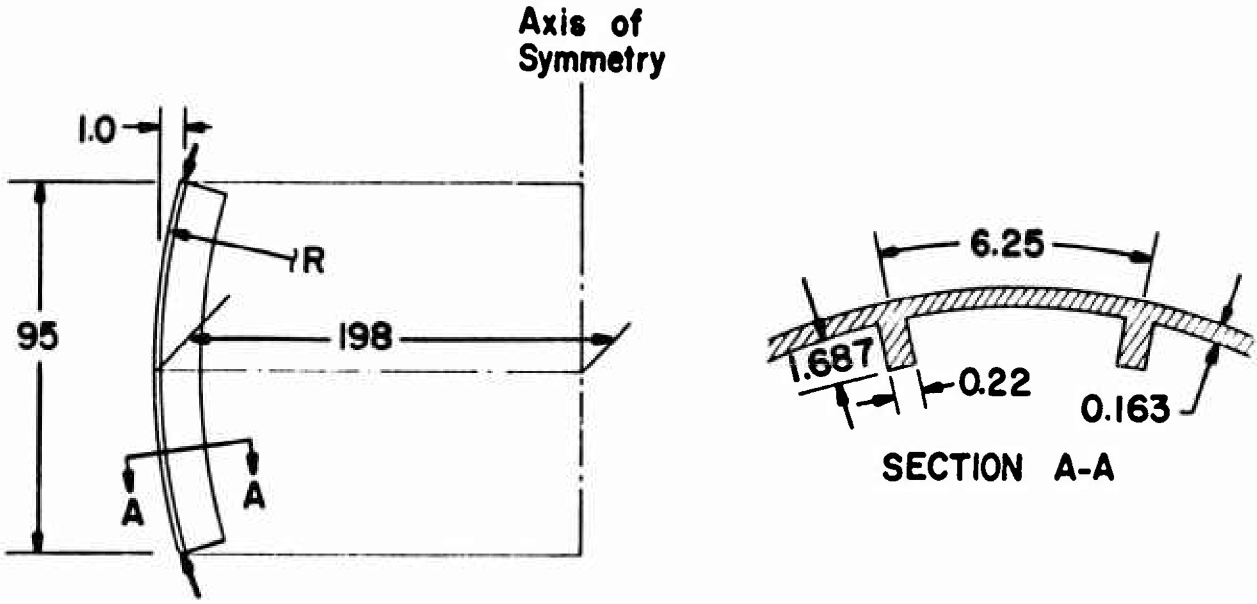


Fig. 7-10 Sample Case 4 – Toroidal Shell

SAMPLE CASE 4
INPUT

```
   SAMPLE CASE 4 - TORUS                                               C-1
      8     1     1                                                    G-1
87.59       90.0        2.5        -930.625  1128.625                  G-2
      9     7                                                          D-1
      1     4     4     4                                              B-1
      1     0     1.                                                   L-1
0.          0.          1.               0      0      2      1     0  L-2
            5340.0            1     1    16                            P-1A
      1     4     4     5                                              P-1A1
      0     0     1     3     0     1     1                            O-1
0.          0.9037      2.41                                           O-2
      2     1                                                          M-1
.163        10000000.  .3          0.0                                 M-2B
10000000.  .3           0.         6.25        0.                      N-1
.22         1.687                                                      N-2A
```

Fig. 7-11 Display of Input Cards for Sample Case 4

SAMPLE CASE 4 – OUTPUT

```
   SAMPLE CASE 4 - TORUS
BUCKLING ANALYSIS.            1 LOAD PATTERNS.

TYPE OF SURFACE IS     TORUS

SURFACE CONSTANTS =  8.7590000E+01, 9.0000000E+01, 2.5000000E+00,-9.3062500E+02, 1.1286250E+03,

 BLANK COMMON ARRAY WORKING SPACE=      15000

  FINITE DIFFERENCE MESH.   9 ROWS.    7 COLUMNS.  MESH SPACING. H=    .3012, K=    .4167

NRW1=  -0, NRW2= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS SIMPLESUPRT.
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =     1

 USER-LOAD FLAG =     0,  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP = -0.        , MAXIMUM LOAD =  1.0000000E+00

      PZ             PY              PX          JZ   JY   JX  ROW  COL
 0.             0.             1.0000000E+00     0    0    2    1    0

BOUNDARY CONDITIONS FOR BUCKLING DISPLACEMENTS
BOUNDARY CONDITION AT LINE 1 IS SIMPLESUPRT.
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  ANTI-METRIC

 ISHIFT   ITERAT    SHIFT
   1        15      5.34300E+03

IPX=    3 IPY=    0 IPRD=    1 IPRS=    1 IPLOT=   -0

IWALL=     2,  NSTRI=    1,  NRING=   -0,  IP=   -0,  IM=  ^-0,  JM=   -0

  AT =  1.6300000E-01  EX1 =  1.0000000E+07 XNU =  3.0000000E-01
   Z =  0.             EY1 =  1.0000000E+07   G =  3.8461538E+06

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CF82
     CCC(I,1)             CCC(I,2)             CCC(I,3)             CCC(I,4)             CCC(I,5)             CCC(I,6)

       1.791209E+06         0.                   0.                   0.                   0.                   0.
       5.373626E+05         1.791209E+06         0.                   0.                   0.                   0.
       0.                   0.                   6.269231E+05         0.                   0.                   0.
       0.                   0.                   0.                   3.965886E+03         0.                   0.
       0.                   0.                   0.                   1.189766E+03         3.965886E+03         0.
       0.                   0.                   0.                   0.                   0.                   1.388060E+03

ANALYSIS IS FOR A SHELL STIFFENED BY STRINGERS ONLY.

INTERNAL STRINGERS.

MODULUS OF ELASTICITY =  1.00000000E+07     POISSON RATIO =  3.00000000E-01     STRINGER SPACING =  6.25000000E+00

RECTANGULAR STIFFENER (STRINGER) DATA.
STRINGER THICKNESS =  2.20000000E-01     STRINGER HEIGHT =  1.68700000E+00
```

Fig. 7-12 Excerpt of Output for Sample Case 4

```
THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE STIFF
     CCC(I,1)            CCC(I,2)            CCC(I,3)            CCC(I,4)            CCC(I,5)            CCC(I,6)

      2.385033E+06        0.                  0.                  0.                  0.                  0.
      5.373626E+05        1.791209E+06        0.                  0.                  0.                  0.
      0.                  0.                  6.269231E+05        0.                  0.                  0.
     -5.492872E+05        0.                  0.                  6.528983E+05        0.                  0.
      0.                  0.                  0.                  1.189766E+03        3.965086E+03        0.
      0.                  0.                  0.                  0.                  0.                  2.389248E+03


 CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=    1.117.  NR OF IO REQUESTS (TAPE2)=    11.      WORDS USED (TAPE2)=   28560.     WORDS TRANSFERRED (TAPE2)=       30608

 FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=    4.246.  NR OF IO REQUESTS (TAPE2)=    22.      WORDS USED (TAPE2)=   42584.     WORDS TRANSFERRED (TAPE2)=       67460

 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=    4.512.  NR OF IO REQUESTS (TAPE2)=    26.      WORDS USED (TAPE2)=   49596.     WORDS TRANSFERRED (TAPE2)=       88496


DETERMINANT OF STIFFNESS MATRIX=   1.6490477E+01*10.**   450.   NUMBER OF NEGATIVE ROOTS =    0
       53 NODES.        297 EQUATIONS.    MAXIMUM BAND WIDTH =          60
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=    5.267.  NR OF IO REQUESTS (TAPE2)=    30.      WORDS USED (TAPE2)=   49596.     WORDS TRANSFERRED (TAPE2)=      103544


LINEAR SOLUTION.  PA=   1.000000E+00,    PB=   0.
ROW  8.  X=      2.109, AXIAL LOAD=       -8.599771E+00,BENDING MOMENT=      0.

  ROW   1,  X=     0.0000
                                OUTER SURFACE                                     INNER SURFACE
      X         Y        SIGMAX      SIGMAY          TAU          SIGMAX      SIGMAY          TAU      SIGMA(STRNGR)  SIGMA(RING)
    0.000    0.000   -4.6059E+00  -1.3318E+00   0.               -4.6062E+00  -1.3351E+00   0.               -4.2060E+00   0.
    0.000     .417   -4.6059E+00  -1.3318E+00   4.0657E-13       -4.6062E+00  -1.3351E+00   4.1146E-13       -4.2060E+00   0.
    0.000     .833   -4.6059E+00  -1.3318E+00   1.3398E-14       -4.6062E+00  -1.3351E+00   1.8381E-14       -4.2060E+00   0.
    0.000    1.250   -4.6059E+00  -1.3318E+00   4.6937E-14       -4.6062E+00  -1.3351E+00   2.6841E-14       -4.2060E+00   0.
    0.000    1.667   -4.6059E+00  -1.3318E+00   1.3228E-13       -4.6062E+00  -1.3351E+00   1.3725E-13       -4.2060E+00   0.
    0.000    2.083   -4.6059E+00  -1.3318E+00  -4.5859E-14       -4.6062E+00  -1.3351E+00  -5.5625E-15       -4.2060E+00   0.
    0.000    2.500   -4.6059E+00  -1.3318E+00   0.               -4.6062E+00  -1.3351E+00   0.               -4.2060E+00   0.


  ROW   9,  X=     2.4100
                                OUTER SURFACE                                     INNER SURFACE
      X         Y        SIGMAX      SIGMAY          TAU          SIGMAX      SIGMAY          TAU      SIGMA(STRNGR)  SIGMA(RING)
    2.410    0.000   -4.2604E+00   1.2225E+00   0.               -4.3245E+00   1.1994E+00   0.               -4.6557E+00   0.
    2.410     .417   -4.2604E+00   1.2225E+00  -5.9334E-23       -4.3245E+00   1.1994E+00   1.6984E-22       -4.6557E+00   0.
    2.410     .833   -4.2604E+00   1.2225E+00  -1.2369E-22       -4.3245E+00   1.1994E+00   3.1232E-22       -4.6557E+00   0.
    2.410    1.250   -4.2604E+00   1.2225E+00  -2.5938E-22       -4.3245E+00   1.1994E+00   5.0890E-22       -4.6557E+00   0.
    2.410    1.667   -4.2604E+00   1.2225E+00  -3.1142E-22       -4.3245E+00   1.1994E+00   5.3995E-22       -4.6557E+00   0.
    2.410    2.083   -4.2604E+00   1.2225E+00  -2.0087E-22       -4.3245E+00   1.1994E+00   3.0368E-22       -4.6557E+00   0.
    2.410    2.500   -4.2604E+00   1.2225E+00   0.               -4.3245E+00   1.1994E+00   0.               -4.6557E+00   0.
CP SECONDS=    6.115.  NR OF IO REQUESTS (TAPE2)=    50.      WORDS USED (TAPE2)=   50188.     WORDS TRANSFERRED (TAPE2)=      176400
```

Fig. 7-12 (Cont.)

ROW 1. X= 0.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	2.265681E-05	9.127910E-07	0.
2	.4167	0.	0.	2.265681E-05	9.127910E-07	0.
3	.8333	0.	0.	2.265681E-05	9.127910E-07	0.
4	1.2500	0.	0.	2.265681E-05	9.127910E-07	0.
5	1.6667	0.	0.	2.265681E-05	9.127910E-07	0.
6	2.0833	0.	-3.851868E-34	2.265681E-05	9.127910E-07	1.953512E-36
7	2.5000	0.	0.	2.265681E-05	9.127910E-07	0.

ROW 4. X= .9037

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	2.766562E-05	0.	1.457937E-05	1.679694E-06	0.
2	.4167	2.766562E-05	-4.341886E-20	1.457937E-05	1.679694E-06	-1.482367E-19
3	.8333	2.766562E-05	-5.885992E-20	1.457937E-05	1.679694E-06	-2.806149E-19
4	1.2500	2.766562E-05	-5.881594E-20	1.457937E-05	1.679694E-06	-8.775545E-19
5	1.6667	2.766562E-05	-5.853377E-20	1.457937E-05	1.679694E-06	-1.193583E-18
6	2.0833	2.766562E-05	-4.632256E-20	1.457937E-05	1.679694E-06	-1.263873E-18
7	2.5000	2.766562E-05	0.	1.457937E-05	1.679694E-06	0.

ROW 9. X= 2.4100

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	4.947361E-05	0.	-1.694066E-21	1.501000E-24	0.
2	.4167	4.947361E-05	-6.034781E-20	-1.694066E-21	1.501000E-24	-7.706834E-19
3	.8333	4.947361E-05	-1.037580E-19	-3.388132E-21	3.002000E-24	-1.471363E-18
4	1.2500	4.947361E-05	-1.372480E-19	-1.694066E-21	1.501000E-24	-2.592631E-18
5	1.6667	4.947361E-05	-1.257030E-19	-3.388132E-21	3.002000E-24	-2.873048E-18
6	2.0833	4.947361E-05	-5.655299E-20	0.	0.	-2.242589E-18
7	2.5000	4.947361E-05	0.	0.	0.	0.

ROW 1. X= 0.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.000342E+00	-2.173530E-01	0.	2.307871E-01	7.238550E-06	0.
2	.4167	-1.000342E+00	-2.173530E-01	6.666897E-14	2.307871E-01	7.238550E-06	-1.880112E-17
3	.8333	-1.000342E+00	-2.173530E-01	2.589962E-15	2.307871E-01	7.238550E-06	-1.835259E-17
4	1.2500	-1.000342E+00	-2.173530E-01	6.012893E-15	2.307871E-01	7.238550E-06	7.402207E-17
5	1.6667	-1.000342E+00	-2.173530E-01	2.195639E-14	2.307871E-01	7.238550E-06	-1.830953E-17
6	2.0833	-1.000342E+00	-2.173530E-01	-4.190884E-15	2.307871E-01	7.238550E-06	-1.484323E-16
7	2.5000	-1.000342E+00	-2.173530E-01	0.	2.307871E-01	7.238550E-06	0.

ROW 4. X= .9037

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-9.975470E-01	1.975268E-02	0.	2.793657E-01	8.031298E-05	0.
2	.4167	-3.975479E-01	1.975268E-02	4.082477E-14	2.793657E-01	8.031298E-05	-9.114550E-18
3	.8333	-9.975470E-01	1.975268E-02	2.822526E-14	2.793657E-01	8.031298E-05	2.110404E-16
4	1.2500	-9.975470E-01	1.975268E-02	8.754309E-14	2.793657E-01	8.031298E-05	6.151522E-16
5	1.6667	-9.975470E-01	1.975268E-02	1.833653E-14	2.793657E-01	8.031298E-05	7.342990E-16
6	2.0833	-9.975470E-01	1.975268E-02	8.033192E-14	2.793657E-01	8.031298E-05	4.6000[illegible]1E-16
7	2.5000	-9.975470E-01	1.975268E-02	0.	2.793657E-01	8.031298E-05	0.

ROW 9. X= 2.4100

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-9.953984E-01	1.973810E-01	0.	2.786256E-01	5.103531E-05	0.
2	.4167	-9.953984E-01	1.973810E-01	8.941357E-24	2.786256E-01	5.103531E-05	-8.412210E-25
3	.8333	-9.953984E-01	1.973810E-01	1.537333E-23	2.786256E-01	5.103531E-05	-1.606030E-24
4	1.2500	-9.953984E-01	1.973810E-01	2.033545E-23	2.786256E-01	5.103531E-05	-2.829924E-24
5	1.6667	-9.953984E-01	1.973810E-01	1.862488E-23	2.786256E-01	5.103531E-05	-3.136007E-24
6	2.0833	-9.953984E-01	1.973810E-01	8.379195E-24	2.786256E-01	5.103531E-05	-2.447844E-24
7	2.5000	-9.953984E-01	1.973810E-01	0.	2.786256E-01	5.103531E-05	0.

Fig. 7-12 (Cont.)

```
ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=   10.843.  NR OF IO REQUESTS (TAPE2)=    77.      WORDS USED (TAPE2)=   50620.     WORDS TRANSFERRED (TAPE2)=      277976


DETERMINANT OF STIFFNESS MATRIX=   1.4234652E+02*10.**    430.   NUMBER OF NEGATIVE ROOTS =   0
     63 NODES.        297 EQUATIONS.    MAXIMUM BAND WIDTH =           60
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=   11.587.  NR OF IO REQUESTS (TAPE2)=    81.      WORDS USED (TAPE2)=   50620.     WORDS TRANSFERRED (TAPE2)=      293024

EIGENVALUE SHIFT=   5.3400000E+03,      NUMBER OF NEGATIVE   ROOTS=        0

ITERATION          EIGENVALUE                          EIGENVALUE
                (RAYLEIGH QUOTIENT)               (ACCELERATED ESTIMATE)
    0            5.7630687E+03                       5.3400000E+03
    1            6.3491999E+03                       4.1782172E+03
    2            5.4023745E+03                       6.4098813E+03
    3            5.3895397E+03                       6.3910572E+03
    4            6.3905345E+03                       6.3903588E+03
    5            6.3903373[illegible]+03                       6.3903742E+03

 THE BUCKLING LOAD BASED ON LINEAR BIFURCATION THEORY IS  6.390374E+03 TIMES THE STARTING LOAD.

  ROW    1,  X=      0.0000
  COL        Y               W                     V                U                 BETAX               BETAY
    1     0.0000     0.                  1.387779E-17    0.                  0.                 -7.038269E-20
    2      .4167     0.                  1.387779E-17   -4.237038E-03       2.559511E-02       -7.038269E-20
    3      .8333     0.                  1.387779E-17   -8.185444E-03       4.944595E-02       -7.038269E-20
    4     1.2500     0.                  6.938894E-18   -1.157597E-02       6.992711E-02       -3.519135E-20
    5     1.6667     0.                  1.387779E-17   -1.417760E-02       8.564286E-02       -7.038269E-20
    6     2.0833     0.                  0.             -1.581306E-02       9.552218E-02        0.
    7     2.5000     0.                  0.             -1.637089E-02       9.889193E-02        0.

  ROW    4.  X=      .9037
  COL        Y               W                     V                U                 BETAX               BETAY
    1     0.0000     0.                  3.088864E-02    0.                  0.                  1.799469E-01
    2      .4167     2.588191E-01        2.983614E-02    1.039871E-03      -3.766918E-03        1.738153E-01
    3      .8333     5.000001E-01        2.675035E-02    2.008877E-03      -7.277124E-03        1.558385E-01
    4     1.2500     7.071069E-01        2.184157E-02    2.840980E-03      -1.029140E-02        1.272416E-01
    5     1.6667     8.660255E-01        1.544432E-02    3.479475E-03      -1.260434E-02        8.997338E-02
    6     2.0833     9.659258E-01        7.994568E-03    3.880850E-03      -1.405831E-02        4.657364E-02
    7     2.5000     1.000000E+00        0.              4.017752E-03      -1.455423E-02        0.

  ROW    9.  X=      [illegible].4100
  COL        Y               W                     V                U                 BETAX               BETAY
    1     0.0000     0.                 -1.010974E-02    0.                  0.                 -5.441881E-02
    2      .4167    -7.843097E-02       -9.765255E-03   -1.734723E-18       1.537024E-21       -5.256455E-02
    3      .8333    -1.515171E-01       -8.755289E-03   -1.734723E-18       1.537024E-21       -4.712813E-02
    4     1.2500    -2.142776E-01       -7.148564E-03   -3.469447E-18       3.074048E-21       -3.848001E-02
    5     1.6567    -2.624355E-01       -5.054870E-03   -3.469447E-18       3.074048E-21       -2.720951E-02
    6     2.0833    -2.927359E-01       -2.615593E-03   -6.938894E-18       6.148095E-21       -1.408469E-02
    7     2.5000    -3.030346E-01        0.              0.                  0.                  0.
CP SECONDS=   13.123.  NR OF IO REQUESTS (TAPE2)=   117.      WORDS USED (TAPE2)=   58144.     WORDS TRANSFERRED (TAPE2)=      445908
```

Fig. 7-12 (Cont.)

7.5 SAMPLE CASE 5 – CORRUGATED CYLINDER

The critical axial line load according to bifurcation theory is to be determined for a simply supported corrugated circular cylindrical shell stiffened with internal rings. Because this is a shell of revolution with axially symmetrical loading, it could be analyzed with the BOSOR4 computer program (Ref. 15) leading to a critical axial line load of 952 lb/in. for a mode with 7 circumferential waves. Therefore, a shell segment covering one half the length and 1/28 of the circumference can be analyzed. That is, the shell segment is expected to buckle with one quarter of a sine wave in the circumferential direction. Thus, the boundary condition on lines 2 and 4 is identical to that of Sample Case 4 both for the prebuckling and for the buckling analysis. The geometry of the shell is shown in Fig. 7-13. The input cards associated with this case are displayed in Fig. 7-14. Portions of the output are presented in Fig. 7-15.

To check the branch for arbitrary stiffeners, the rectangular rings with 4.0-in. spacing were described in the input as arbitrary rings, and to reduce the number of iterations, an initial eigenvalue shift at 1000.0 was utilized. The critical axial line load determined here is 987 lb/in.

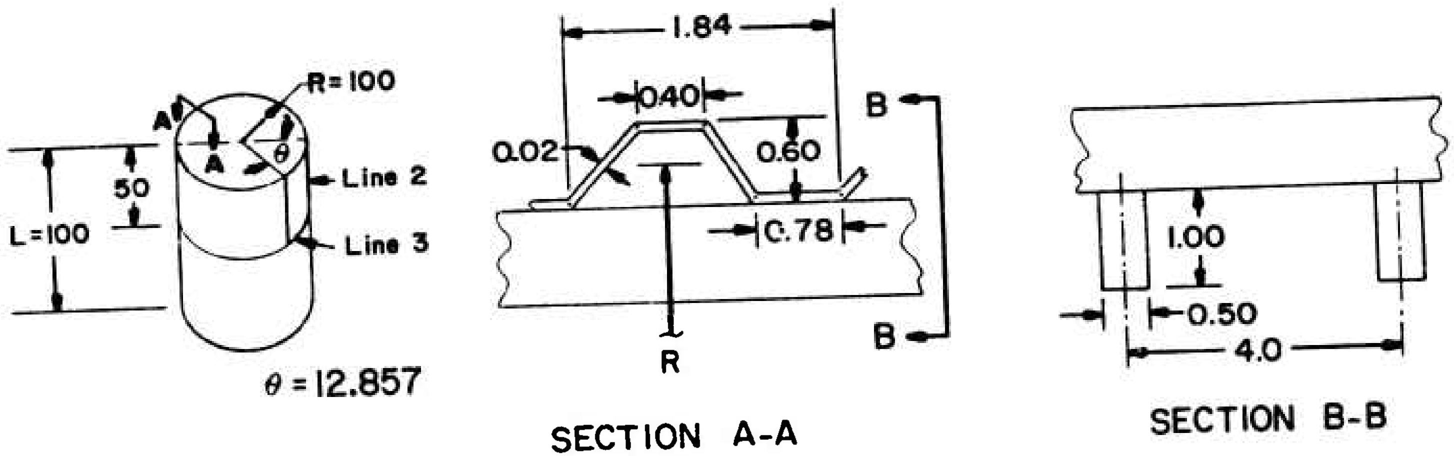


Fig. 7-13 Sample Case 5 – Corrugated Cylinder

SAMPLE CASE 5
INPUT

```
  SAMPLE CASE 5 - CORRUGATED CYLINDER                                              C-1
   1     1     1                                                                   G-1
50.0      12.857     100.0                                                         G-2
  31     7                                                                         D-1
   1     4     4     4                                                             B-1
   1     0     1.                                                                  L-1
0.       0.          1.           0      0      2      1      0                    L-2
         1000.0           1     2    20                                            P-1A
   1     4     4     5                                                             P-1A1
   0     0     1     3     0      1      1                                         O-1
0.       15.         25.          50.                                              O-2
   6     0     1                                                                   M-1
0.02      10000000. 0.3          0.4         0.6         0.78        1.84      0.0 M-2F
10000000.00.3       1.0          4.0         0.0                                   N-3
0.5       0.041     0.0          157092.3    0.5         1.0                       N-4B
```

Fig. 7-14 Display of Input Cards for Sample Case 5

SAMPLE CASE 5 – OUTPUT

```
   SAMPLE CASE 5 - CORRUGATED CYLINDER
BUCKLING ANALYSIS.            1 LOAD PATTERNS.

TYPE OF SURFACE IS CYLINDER

SURFACE CONSTANTS =  5.0000000E+01, 1.2857000E+01, 1.0000000E+02,

 BLANK COMMON ARRAY WORKING SPACE=     15000

  FINITE DIFFERENCE MESH.  31 ROWS,    7 COLUMNS.  MESH SPACING. H=    1.6667, K=   2.1428

NRW1=  -0, NRW2= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS SIMPLESUPRT.
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =     1

 USER-LOAD FLAG =      0.  STARTING LOAD FACTOR =  1.0000000E+00.  LOAD STEP = -0.          ,  MAXIMUM LOAD =  1.0000000E+00

      PZ             PY             PX            JZ    JY    JX   ROW  COL
 0.             0.           1.0000000E+00        0     0     2     1    0

BOUNDARY CONDITIONS FOR BUCKLING DISPLACEMENTS
BOUNDARY CONDITION AT LINE 1 IS SIMPLESUPRT.
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  ANTI-METRIC

 ISHIFT    ITERAT    SHIFT
   2         20        1.00000E+03

IPX=     3 IPY=     0 IPRD=     1 IPRS=     1 IPLOT=    -0

IWALL=     6,  NSTRI=    0,  NRING=    1,  IP=   -0,  IM=   -0,  JM=   -0

ANALYSIS IS FOR A SHELL WITH CORRUGATED SKIN.
.
MODULUS OF ELASTICITY =  1.000000000E+07     POISSON RATIO =  3.000000000E-01

C =  4.000000000E-01     H =  6.000000000E-01  °  D =  7.000000000E-01     T =  2.000000000E-02     B =  1.000000000E+00
Z = 0.

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CFB6
      CCC(I,1)             CCC(I,2)              CCC(I,3)              CCC(I,4)              CCC(I,5)              CCC(I,6)

       2.733278E+05         0.                    0.                    0.                    0.                    0.
       0.                   0.                    0.                    0.                    0.                    0.
       0.                   0.                    5.628631E+04          0.                    0.                    0.
      -1.197826E+04         0.                    0.                    1.486748E+04          0.                    0.
       0.                   0.                    0.                    0.                    5.360601E+00          0.
       0.                   0.                   -3.371061E+03          0.                    0.                    2.028393E+02

ANALYSIS IS FOR A SHELL STIFFENED BY RINGS ONLY.

INTERNAL RINGS.

MODULUS OF ELASTICITY =  1.000000000E+07     POISSON RATIO =  3.000000000E-01     RING SPACING =  4.000000000E+00
```

Fig. 7-15 Excerpt of Output for Sample Case 5

```
ARBITRARY STIFFENER (RING) DATA.
              AREA OF        MOMENT OF        MOMENT OF          TORSIONAL
           CROSS-SECTION   INERTIA,IX       INERTIA,IZ         STIFFNESS      ECCENTRICITY    RING HEIGHT

            5.000000E-01   4.100000E-02    0.                 1.576923E+05    5.000000E-01    1.000000E+00

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE STIFF
     CCC(I,1)             CCC(I,2)             CCC(I,3)             CCC(I,4)             CCC(I,5)             CCC(I,6)

      2.733275E+05         0.                   0.                   0.                   0.                   0.
      0.                   1.250000E+06         0.                   0.                   0.                   0.
      0.                   0.                   5.628631E+04         0.                   0.                   0.
     -1.197826E+04         0.                   0.                   1.406748E+06         0.                   0.
      0.                  -1.000000E+06         0.                   0.                   9.025054E+05         0.
      0.                   0.                  -3.371061E+03         0.                   0.                   1.005860E+04

 CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=     2.616.   NR OF IO REQUESTS (TAPE2)=     15.      WORDS USED (TAPE2)=   51464.     WORDS TRANSFERRED (TAPE2)=       55560

 FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=     5.435.   NR OF IO REQUESTS (TAPE2)=     35.      WORDS USED (TAPE2)=   94831.     WORDS TRANSFERRED (TAPE2)=      146390

 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=     6.445.   NR OF IO REQUESTS (TAPE2)=     49.      WORDS USED (TAPE2)=  121075.     WORDS TRANSFERRED (TAPE2)=      228099

DETERMINANT OF STIFFNESS MATRIX=    1.7663381E+01*10.**   1528.    NUMBER OF NEGATIVE ROOTS =    0
      217 NODES.        891 EQUATIONS.    MAXIMUM BAND WIDTH =          60
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=     9.159.   NR OF IO REQUESTS (TAPE2)=     59.      WORDS USED (TAPE2)=  121075.     WORDS TRANSFERRED (TAPE2)=      282635

LINEAR SOLUTION.  PA=    1.000000E+00.    PB=    0.
ROW 30.   X=      48.333.  AXIAL LOAD=        -2.243970E+01.BENDING MOMENT=       0.
                        AXIAL  STRESS
      X          Y      OUTER  SURFACE    INNER  SURFACE    SIGMA(STRNGR)    SIGMA(RING)
    0.000     0.000       -3.6586E+01       -3.6586E+01      0.                -1.9722E-26
    0.000     2.143       -3.6586E+01       -3.6586E+01      0.                  0.
    0.000     4.286       -3.6586E+01       -3.6586E+01      0.                -9.8608E-27
    0.000     6.428       -3.6586E+01       -3.6586E+01      0.                  1.9722E-26
    0.000     8.571       -3.6586E+01       -3.6586E+01      0.                -3.9443E-26
    0.000    10.714       -3.6586E+01       -3.6586E+01      0.                  0.
    0.000    12.857       -3.6586E+01       -3.6586E+01      0.                  0.

                        AXIAL  STRESS
      X          Y      OUTER  SURFACE    INNER  SURFACE    SIGMA(STRNGR)    SIGMA(RING)
   15.000     0.000       -3.6189E+01       -3.6882E+01      0.                 7.8161E-03
   15.000     2.143       -3.6189E+01       -3.6882E+01      0.                 7.8161E-03
   15.000     4.286       -3.6189E+01       -3.6882E+01      0.                 7.8161E-03
   15.000     6.428       -3.6189E+01       -3.6882E+01      0.                 7.8161E-03
   15.000     8.571       -3.6189E+01       -3.6882E+01      0.                 7.8161E-03
   15.000    10.714       -3.6189E+01       -3.6882E+01      0.                 7.8161E-03
   15.000    12.857       -3.6189E+01       -3.6882E+01      0.                 7.8161E-03
```

Fig. 7-15 (Cont.)

X	Y	AXIAL STRESS OUTER SURFACE	INNER SURFACE	SIGMA(STRNGR)	SIGMA(RING)
41.667	0.000	-3.6585E+01	-3.6587E+01	0.	-1.8209E-04
41.667	2.143	-3.6585E+01	-3.6587E+01	0.	-1.8209E-04
41.667	4.286	-3.6585E+01	-3.6587E+01	0.	-1.8209E-04
41.667	6.428	-3.6585E+01	-3.6587E+01	0.	-1.8209E-04
41.667	8.571	-3.6585E+01	-3.6587E+01	0.	-1.8209E-04
41.667	10.714	-3.6585E+01	-3.6587E+01	0.	-1.8209E-04
41.667	12.857	-3.6585E+01	-3.6587E+01	0.	-1.8209E-04

X	Y	AXIAL STRESS OUTER SURFACE	INNER SURFACE	SIGMA(STRNGR)	SIGMA(RING)
43.333	0.000	-3.6585E+01	-3.6587E+01	0.	-3.6110E-05
43.333	2.143	-3.6585E+01	-3.6587E+01	0.	-3.6110E-05
43.333	4.286	-3.6585E+01	-3.6587E+01	0.	-3.6110E-05
43.333	6.428	-3.6585E+01	-3.6587E+01	0.	-3.6110E-05
43.333	8.571	-3.6585E+01	-3.6587E+01	0.	-3.6110E-05
43.333	10.714	-3.6585E+01	-3.6587E+01	0.	-3.6110E-05
43.333	12.857	-3.6585E+01	-3.6587E+01	0.	-3.6110E-05

X	Y	AXIAL STRESS OUTER SURFACE	INNER SURFACE	SIGMA(STRNGR)	SIGMA(RING)
45.000	0.000	-3.6586E+01	-3.6586E+01	0.	4.7910E-05
45.000	2.143	-3.6586E+01	-3.6586E+01	0.	4.7910E-05
45.000	4.286	-3.6586E+01	-3.6586E+01	0.	4.7910E-05
45.000	6.428	-3.6586E+01	-3.6586E+01	0.	4.7910E-05
45.000	8.571	-3.6586E+01	-3.6586E+01	0.	4.7910E-05
45.000	10.714	-3.6586E+01	-3.6586E+01	0.	4.7910E-05
45.000	12.857	-3.6586E+01	-3.6586E+01	0.	4.7910E-05

X	Y	AXIAL STRESS OUTER SURFACE	INNER SURFACE	SIGMA(STRNGR)	SIGMA(RING)
46.667	0.000	-3.6586E+01	-3.6586E+01	0.	9.0360E-05
46.667	2.143	-3.6586E+01	-3.6586E+01	0.	9.0360E-05
46.667	4.286	-3.6586E+01	-3.6586E+01	0.	9.0360E-05
46.667	6.428	-3.6586E+01	-3.6586E+01	0.	9.0360E-05
46.667	8.571	-3.6586E+01	-3.6586E+01	0.	9.0360E-05
46.667	10.714	-3.6586E+01	-3.6586E+01	0.	9.0360E-05
46.667	12.857	-3.6586E+01	-3.6586E+01	0.	9.0360E-05

X	Y	AXIAL STRESS OUTER SURFACE	INNER SURFACE	SIGMA(STRNGR)	SIGMA(RING)
48.333	0.000	-3.6586E+01	-3.6586E+01	0.	1.0841E-04
48.333	2.143	-3.6586E+01	-3.6586E+01	0.	1.0841E-04
48.333	4.286	-3.6586E+01	-3.6586E+01	0.	1.0841E-04
48.333	6.428	-3.6586E+01	-3.6586E+01	0.	1.0841E-04
48.333	8.571	-3.6586E+01	-3.6586E+01	0.	1.0841E-04
48.333	10.714	-3.6586E+01	-3.6586E+01	0.	1.0841E-04
48.333	12.857	-3.6586E+01	-3.6586E+01	0.	1.0841E-04

X	Y	AXIAL STRESS OUTER SURFACE	INNER SURFACE	SIGMA(STRNGR)	SIGMA(RING)
50.000	0.000	-3.6586E+01	-3.6586E+01	0.	1.1316E-04
50.000	2.143	-3.6586E+01	-3.6586E+01	0.	1.1316E-04
50.000	4.286	-3.6586E+01	-3.6586E+01	0.	1.1316E-04
50.000	6.428	-3.6586E+01	-3.6586E+01	0.	1.1316E-04
50.000	8.571	-3.6586E+01	-3.6586E+01	0.	1.1316E-04
50.000	10.714	-3.6585E+01	-3.6585E+01	0.	1.1316E-04
50.000	12.857	-3.6585E+01	-3.6585E+01	0.	1.1315E-04

CP SECONDS= 11.482. NR OF IO REQUESTS (TAPE2)= 97. WORDS USED (TAPE2)= 122099. WORDS TRANSFERRED (TAPE2)= 465831

Fig. 7-15 (Cont.)

```
ROW   1.  X=   0.0000
COL        Y          W           V               U             BETAX          BETAY
  1    0.0000     0.          0.            1.031390E-04   -4.757824E-06    0.
  2    2.1428     0.          0.            1.031390E-04   -4.757824E-06    0.
  3    4.2857     0.          0.            1.031390E-04   -4.757824E-06    0.
  4    6.4285     0.          0.            1.031390E-04   -4.757824E-06    0.
  5    8.5713     0.          0.            1.031390E-04   -4.757824E-06    0.
  6   10.7142     0.          0.            1.031390E-04   -4.757824E-06    0.
  7   12.8570     0.          0.            1.031390E-04   -4.757824E-06    0.

ROW  10.  X=  15.0000
COL        Y          W              V              U             BETAX          BETAY
                                   0.                                           0.
  1    0.0000   7.016111E-08   -1.600962E-10   1.280400E-04   2.593953E-07   3.379691E-10
  2    2.1428   7.016111E-08   -1.494594E-10   1.280400E-04   2.593953E-07   4.177943E-10
  3    4.2857   7.016111E-08   -1.129623E-10   1.280400E-04   2.593953E-07   4.147934E-10
  4    6.4285   7.016111E-08   -6.798439E-19   1.280400E-04   2.593953E-07   3.314380E-10
  5    8.5713   7.016111E-08   -2.022359E-19   1.280400E-04   2.593953E-07   1.677102E-10
  6   10.7142   7.016111E-08    0.             1.280400E-04   2.593953E-07   0.
  7   12.8570   7.016111E-08                   1.280400E-04   2.593953E-07

ROW  16.  X=  25.0000
COL        Y          W              V              U             BETAX          BETAY
                                   0.                                           0.
  1    0.0000   1.128240E-07   -1.193871E-10   9.146709E-05   -4.191380E-08   3.103473E-10
  2    2.1428   1.128240E-07   -1.143343E-10   9.146709E-05   -4.191380E-08   4.269095E-10
  3    4.2857   1.128240E-07   -6.031079E-19   9.146709E-05   -4.191380E-08   4.213943E-10
  4    6.4285   1.128240E-07   -1.633334E-19   9.146709E-05   -4.191380E-08   3.191376E-10
  5    8.5713   1.128240E-07   -4.110166E-19   9.146709E-05   -4.191380E-08   1.229365E-10
  6   10.7142   1.128240E-07    0.             9.146709E-05   -4.191380E-08   0.
  7   12.8570   1.128240E-07                   9.146709E-05   -4.191380E-08

ROW   1.  X=   0.0000
COL        Y          MX              MY              MXY             MX'            MY             MXY
                                                  0.                             1.954393E-27    0.
  1    0.0000   -1.000000E+00   -2.445469E-27    5.456060E-13   4.382379E-02   0.             -4.236400E-14
  2    2.1428   -1.000000E+00    0.              6.466077E-14   4.382379E-02   9.771767E-28   -1.093034E-14
  3    4.2857   -1.000000E+00   -1.222734E-27   -3.149178E-13   4.382379E-02   -1.954393E-27   3.219553E-15
  4    6.4285   -1.000000E+00    2.445469E-27   -7.230243E-13   4.382379E-02   3.908787E-27    2.479019E-14
  5    8.5713   -1.000000E+00   -4.890938E-27   -9.893290E-13   4.382379E-02   0.              5.085989E-14
  6   10.7142   -1.000000E+00    0.              0.              4.382379E-02   0.              0.
  7   12.8570   -1.000000E+00    0.                              4.382379E-02

ROW  10.  X=  15.0000
COL        Y          MX              MY              MXY             MX             MY             MXY
                                                  0.                                             0.
  1    0.0000   -1.000000E+00    9.770139E-04    9.202027E-13   4.547822E-02   -7.016111E-04   -3.262986E-14
  2    2.1428   -1.000000E+00    9.770139E-04    1.449899E-14   4.547822E-02   -7.016111E-04   -9.050647E-16
  3    4.2857   -1.000000E+00    9.770139E-04   -3.576401E-13   4.547822E-02   -7.016111E-04    2.379914E-14
  4    6.4285   -1.000000E+00    9.770139E-04   -6.206346E-13   4.547822E-02   -7.016111E-04    4.009000E-14
  5    8.5713   -1.000000E+00    9.770139E-04   -1.026942E-12   4.547822E-02   -7.016111E-04    6.395090E-14
  6   10.7142   -1.000000E+00    9.770139E-04    0.              4.547822E-02   -7.016111E-04    0.
  7   12.8570   -1.000000E+00    9.770139E-04                    4.547822E-02   -7.016111E-04

ROW  16.  X=  25.0000
COL        Y          MX              MY              MXY             MX             MY             MXY
                                                  0.                                             0.
  1    0.0000   -1.000000E+00    1.410300E-03    3.986321E-13   4.371390E-02   -1.128240E-03   -2.196369E-14
  2    2.1428   -1.000000E+00    1.410300E-03   -1.329279E-14   4.371390E-02   -1.128240E-03    1.099699E-15
  3    4.2857   -1.000000E+00    1.410300E-03   -2.920084E-13   4.371390E-02   -1.128240E-03    1.605495E-14
  4    6.4285   -1.000000E+00    1.410300E-03   -5.073536E-13   4.371390E-02   -1.128240E-03    3.111907E-14
  5    8.5713   -1.000000E+00    1.410300E-03   -7.493000E-13   4.371390E-02   -1.128240E-03    4.473377E-14
  6   10.7142   -1.000000E+00    1.410300E-03    0.              4.371390E-02   -1.128240E-03    0.
  7   12.8570   -1.000000E+00    1.410300E-03                    4.371390E-02   -1.128240E-03
```

Fig. 7-15 (Cont.)

```
THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE STIFF
     CCC(I,1)            CCC(I,2)            CCC(I,3)            CCC(I,4)            CCC(I,5)            CCC(I,6)

      2.733278E+05        0.                  0.                  0.                  0.                  0.
      0.                  1.250000E+06        0.                  0.                  0.                  0.
      0.                  0.                  5.626631E+04        0.                  0.                  0.
     -1.197826E+04        0.                  0.                  1.466746E+04        0.                  0.
      0.                 -1.000000E+06        0.                  0.                  9.025054E+05        0.
      0.                  0.                 -3.371061E+03        0.                  0.                  1.085060E+04

 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=   18.437.  NR OF IO REQUESTS (TAPE2)=   147.      WORDS USED (TAPE2)=  123123.     WORDS TRANSFERRED (TAPE2)=      785084


DETERMINANT OF STIFFNESS MATRIX=  -4.5140842E+07*10.**  1430.   NUMBER OF NEGATIVE ROOTS =   1
      217 NODES,        891 EQUATIONS.   MAXIMUM BAND WIDTH =          60
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=   20.986.  NR OF IO REQUESTS (TAPE2)=   157.      WORDS USED (TAPE2)=  123123.     WORDS TRANSFERRED (TAPE2)=      759540

EIGENVALUE SHIFT=   1.0000000E+03,     NUMBER OF NEGATIVE   ROOTS=       1

ITERATION          EIGENVALUE                     EIGENVALUE
                (RAYLEIGH QUOTIENT)           (ACCELERATED ESTIMATE)
    0            9.9702300E+02                  1.0000000E+03
    1            9.8531873E+02                  1.0018159E+03
    2            9.8669629E+02                  9.8655123E+02
    3            9.8649803E+02                  9.8651602E+02
    4            9.8653991E+02                  9.8657627E+02
    5            9.8658766E+02                  9.8660109E+02

 THE BUCKLING LOAD BASED ON LINEAR BIFURCATION THEORY IS  9.866011E+02 TIMES THE STARTING LOAD.

  ROW   1.  X=     0.0000
  COL       Y              W                   V              U                BETAX            BETAY
    1     0.0000      0.                  6.938894E-18   0.                0.               -6.938894E-20
    2     2.1428      0.                  1.387779E-17  -4.305517E-03     2.641020E-02     -1.387779E-19
    3     4.2857      0.                  6.938894E-18  -8.317621E-03     5.102860E-02     -6.938894E-20
    4     6.4285      0.                  6.938894E-18  -1.176209E-02     7.215402E-02     -6.938894E-20
    5     8.5713      0.                  3.469447E-18  -1.440654E-02     8.837026E-02     -3.469447E-20
    6    10.7142      0.                  0.            -1.606841E-02     9.856422E-02      0.
    7    12.8570      0.                  0.            -1.663524E-02     1.020412E-01      0.

  ROW  13.  X=    15.0000
  COL       Y              W                   V              U                BETAX            BETAY
    1     0.0000      0.                  8.072934E-02   0.                0.                6.839659E-02
    2     2.1428      2.580190E-01        7.797856E-02  -4.982191E-04     1.700141E-03      6.605604E-02
    3     4.2857      5.000000E-01        6.991366E-02  -9.624854E-04     3.299074E-03      5.923319E-02
    4     6.4285      7.071368E-01        5.708427E-02  -1.361168E-03     4.666727E-03      4.836369E-02
    5     8.5713      8.660254E-01        4.036467E-02  -1.667074E-03     5.715558E-03      3.419830E-02
    6    10.7142      9.659258E-01        2.089429E-02  -1.859379E-03     6.374867E-03      1.770234E-02
    7    12.8570      1.000000E+00        0.            -1.924971E-03     6.599740E-03      0.

  ROW  16.  X=    25.0000
  COL       Y              W                   V              U                BETAX            BETAY
    1     0.0000      0.                  5.278802E-02   0.                0.                4.407750E-02
    2     2.1428      1.660219E-01        5.090932E-02   2.933736E-03    -1.804495E-02      4.257560E-02
    3     4.2857      3.222751E-01        4.571577E-02   5.667543E-03    -3.486016E-02      3.817223E-02
    4     6.4285      4.557658E-01        3.732677E-02   8.015116E-03    -4.929971E-02      3.116750E-02
    5     8.5713      5.581959E-01        2.639431E-02   9.816472E-03    -6.037957E-02      2.203475E-02
    6    10.7142      6.225877E-01        1.366255E-02   1.094885E-02    -6.734466E-02      1.140810E-02
    7    12.8570      6.445502E-01        0.             1.133509E-02    -6.972032E-02      0.
CP SECONDS=   25.335.  NR OF IO REQUESTS (TAPE2)=   259.      WORDS USED (TAPE2)=  143830.     WORDS TRANSFERRED (TAPE2)=     1333419
```

Fig. 7-15 (Cont.)

7.6 SAMPLE CASE 6 – ELLIPSOID

The critical external pressure according to bifurcation theory is to be determined for an orthotropic ellipsoid shell with clamped circular boundaries. The geometry of the shell is shown in Fig. 7-16.

Analysis with BOSOR4 (Ref. 15) shows that for this shell the critical external pressure is 4327 psi for a mode with 2 circumferential waves. Therefore, a shell covering 1/8 of the circumference can be analyzed.

The meridional boundary conditions are the same as for Case 4. The input cards associated with this case are displayed in Fig. 7-17. Portions of the output are presented in Fig. 7-18.

The critical external pressure determined here is 4741 psi. The discrepancy in critical external pressure is due to the coarseness of the grid used here.

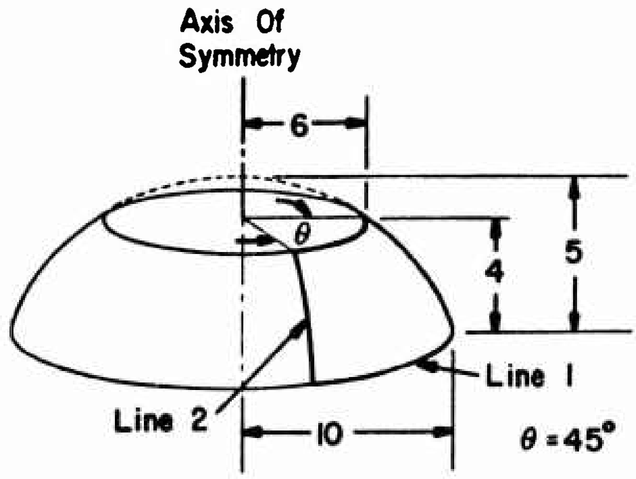


Fig. 7-16 Sample Case 6 – Orthotropic Ellipsoid

SAMPLE CASE 6
INPUT

```
     SAMPLE  CASE  6  -  ELLIPSOID ORTHOTROPIC                      C-1
      7     1     1                                                 G-1
0.0         33.667     45.0       10.0        5.0                   G-2
     16     6                                                       D-1
      2     4     2     4                                           B-1
      1     0    1.                                                 L-1
-1.         0.         0.              4     0     0     0     0    L-2
                             1     2    20                          P-1A
      2     4     2     5                                           P-1A1
      0     0     1     4     0     1     1                         O-1
0.          11.        22.        33.                               O-2
      5                                                             M-1
0.0               1                                                 M-2E1
0.2         10000000. 5000000.  0.1         2000000.                M-2E2
```

Fig. 7-17 Display of Input Cards for Sample Case 6

```
   SAMPLE CASE 6 - ELLIPSOID ORTHOTROPIC
BUCKLING ANALYSIS.          1 LOAD PATTERNS.

TYPE OF SURFACE IS ELLIPSOID

SURFACE CONSTANTS =  0.           , 3.3667000E+01, 4.5000000E+01, 1.0000000E+01, 5.0000000E+00,

 BLANK COMMON ARRAY WORKING SPACE=      15000

  FINITE DIFFERENCE MESH.  16 ROWS,     6 COLUMNS.  MESH SPACING. H=   2.2445, K=   9.0000

NRW1=  -0, NRW2= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS   CLAMPED
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS   CLAMPED
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =      1

 USER-LOAD FLAG =      0.  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP = -0.          , MAXIMUM LOAD =  1.0000000E+00

      PZ             PY            PX          JZ   JY   JX  ROW  COL
-1.0000000E+00 0.              0.           4    0    0    0    0

BOUNDARY CONDITIONS FOR BUCKLING DISPLACEMENTS
BOUNDARY CONDITION AT LINE 1 IS   CLAMPED
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS   CLAMPED
BOUNDARY CONDITION AT LINE 4 IS  ANTI-METRIC

 ISHIFT   ITERAT   SHIFT
   2       20      -0.

IPX=     4 IPY=     0 IPRO=     1 IPRS=     1 IPLOT=     -0 ,

IWALL=     5, NSTRI=   -0, NRING=   -0, IP=   -0, IM=   -0, JM=   -0.

ANALYSIS IS FOR A LAYERED SHELL.
         THERE ARE  1  LAYERS Z = 0.

                                        SHEAR          YOUNGS          YOUNGS          POISSON
                   LAYER     THICKNESS    MODULUS      MODULUS(X)      MODULUS(Y)      RATIO(XY)
                     1   2.0000000E-01 2.0000000E+06 1.0000000E+07 5.0000000E+06 1.0000000E-01

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CFBS
     CCC(I,1)            CCC(I,2)            CCC(I,3)            CCC(I,4)            CCC(I,5)            CCC(I,6)

    2.040816E+06        0.                  0.                  0.                  0.                  0.
    2.040816E+05        1.020408E+06        0.                  0.                  0.                  0.
    0.                  0.                  4.000000E+05        0.                  0.                  0.
    0.                  0.                  0.                  6.802721E+03        0.                  0.
    0.                  0.                  0.                  6.802721E+02        3.401361E+03        0.
    0.                  0.                  0.                  0.                  0.                  1.333333E+03
```

Fig. 7-18 Excerpt of Output for Sample Case 6

```
 CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=     1.736.   NR OF IO REQUESTS (TAPE2)=     14.       WORDS USED (TAPE2)=    48457.      WORDS TRANSFERRED (TAPE2)=        50505
 FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=     6.802.   NR OF IO REQUESTS (TAPE2)=     28.       WORDS USED (TAPE2)=    69878.      WORDS TRANSFERRED (TAPE2)=       107201
 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=     7.315.   NR OF IO REQUESTS (TAPE2)=     36.       WORDS USED (TAPE2)=    82996.      WORDS TRANSFERRED (TAPE2)=       155146

DISPLACEMENT COMPONENT V      X=    3.2545E+01       Y=    4.9500E+01,   HAS BEEN ELIMINATED
  THE INITIAL MAIN DIAGONAL VALUE =  1.8944441E+06,   THE FINAL VALUE =  2.4810433E-06

DETERMINANT OF STIFFNESS MATRIX=   1.1029017E+03*10.**    780.    NUMBER OF NEGATIVE ROOTS =   0
        96 NODES.          432 EQUATIONS.    MAXIMUM BAND WIDTH =          54
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=     8.330.   NR OF IO REQUESTS (TAPE2)=     42.       WORDS USED (TAPE2)=    82996.      WORDS TRANSFERRED (TAPE2)=       184886


LINEAR SOLUTION.  PA=    1.000000E+00.    PB=    0.
ROW 15.   X=      31.423, AXIAL LOAD=         -2.563750E+01,BENDING MOMENT=       0.
    X          Y.  LAYER       SIGMAX             SIGMAY              TAU                SIGMA(STRNGR)   SIGMA(RING)
   0.000     0.000   1        8.8641E+00         7.9197E-01        0.
   0.000     0.000   1       -4.7411E+01        -4.6467E+00        0.
   0.000     9.000   1        8.8641E+00         7.9197E-01       -8.6662E-13
   0.000     9.000   1       -4.7411E+01        -4.6467E+00       -9.0199E-13
   0.000    18.000   1        8.8641E+00         7.9197E-01       -4.7219E-13
   0.000    18.000   1       -4.7411E+01        -4.6467E+00       -4.9147E-13
   0.000    27.000   1        8.8641E+00         7.9197E-01        2.6395E-13
   0.000    27.000   1       -4.7411E+01        -4.6467E+00        2.7473E-13
   0.000    36.000   1        8.8641E+00         7.9197E-01        9.2445E-13
   0.000    36.000   1       -4.7411E+01        -4.6467E+00        9.6218E-13
   0.000    45.000   1        8.8641E+00         7.9197E-01        0.
   0.000    45.000   1       -4.7411E+01        -4.6467E+00        0.


    X          Y   LAYER       SIGMAX             SIGMAY              TAU                SIGMA(STRNGR)   SIGMA(RING)
  33.667     0.000   1       -1.0680E+02        -1.0758E+01        0.
  33.667     0.000   1        5.2482E+01         5.3261E+00        0.
  33.667     9.000   1       -1.0680E+02        -1.0758E+01       -3.5586E-14
  33.667     9.000   1        5.2482E+01         5.3261E+00       -3.6429E-14
  33.667    18.000   1       -1.0680E+02        -1.0758E+01       -3.0795E-13
  33.667    18.000   1        5.2482E+01         5.3261E+00       -3.1525E-13
  33.667    27.000   1       -1.0680E+02        -1.0758E+01        5.9277E-13
  33.667    27.000   1        5.2482E+01         5.3261E+00        6.0681E-13
  33.667    36.000   1       -1.0690E+02        -1.0758E+01       -1.8016E-13
  33.667    36.000   1        5.2482E+01         5.3261E+00       -1.8443E-13
  33.667    45.000   1       -1.0650E+02        -1.0758E+01        0.
  33.667    45.000   1        5.2482E+01         5.3261E+00        0.
CP SECONDS=     9.721.   NR OF IO REQUESTS (TAPE2)=     78.       WORDS USED (TAPE2)=    83508.      WORDS TRANSFERRED (TAPE2)=       318548
```

Fig. 7-18 (Cont.)

ROW 1. X= 0.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	0.	0.	0.
2	9.0000	0.	0.	0.	0.	0.
3	18.0000	0.	0.	-4.235165E-22	1.694066E-22	0.
4	27.0000	0.	0.	-8.470329E-22	3.388132E-22	0.
5	36.0000	0.	3.851860E-34	-4.235165E-22	1.694066E-22	-3.851860E-35
6	45.0000	0.	0.	-4.235165E-22	1.694066E-22	0.

ROW 6. X= 11.2223

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-7.918458E-06	0.	-4.889437E-06	-1.869901E-05	0.
2	9.0000	-7.918458E-06	1.426742E-19	-4.889437E-06	-1.869901E-05	1.222404E-18
3	18.0000	-7.918453E-06	6.833021E-20	-4.889437E-06	-1.869901E-05	7.569879E-19
4	27.0000	-7.918458E-06	-1.149112E-19	-4.889437E-06	-1.869901E-05	-4.443189E-19
5	36.0000	-7.918458E-06	-1.540938E-19	-4.889437E-06	-1.869901E-05	-2.172051E-19
6	45.0000	-7.918458E-06	0.	-4.889437E-06	-1.869901E-05	0.

ROW 11. X= 22.4447

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-3.888192E-05	0.	1.921449E-07	6.043821E-06	0.
2	9.0000	-3.888192E-05	2.121492E-19	1.921449E-07	6.043821E-06	-4.967684E-18
3	18.0000	-3.888192E-05	1.571220E-19	1.921449E-07	6.043821E-06	-6.864805E-19
4	27.0000	-3.888192E-05	-3.081226E-19	1.921449E-07	6.043821E-06	-2.005772E-18
5	36.0000	-3.888192E-05	-2.812468E-19	1.921449E-07	6.043821E-06	1.481056E-18
6	45.0000	-3.888192E-05	0.	1.921449E-07	6.043821E-06	0.

ROW 15. X= 33.6670

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	-4.235165E-22	3.397232E-23	0.
2	9.0000	0.	0.	-4.235165E-22	3.397232E-23	0.
3	18.0000	0.	0.	-8.470329E-22	6.794464E-23	0.
4	27.0000	0.	1.444447E-34	0.	0.	-8.454736E-36
5	36.0000	0.	0.	-4.235165E-22	3.397232E-23	0.
6	45.0000	0.	0.	-4.235165E-22	3.397232E-23	0.

ROW 1. X= 0.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-3.854696E+00	-3.854696E-01	0.	-1.875838E-01	-1.812878E-02	0.
2	9.0000	-3.854696E+00	-3.854696E-01	-1.768618E-13	-1.875838E-01	-1.812878E-02	-1.179078E-16
3	18.0000	-3.854696E+00	-3.854696E-01	-9.636591E-14	-1.875838E-01	-1.812878E-02	-6.424394E-17
4	27.0000	-3.854696E+00	-3.854696E-01	5.386793E-14	-1.875838E-01	-1.812878E-02	3.591195E-17
5	36.0000	-3.854696E+00	-3.854696E-01	1.886636E-13	-1.875838E-01	-1.812878E-02	1.257757E-16
6	45.0000	-3.854696E+00	-3.854696E-01	0.	-1.875838E-01	-1.812878E-02	0.

ROW 6. X= 11.2223

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-4.242643E+00	-7.645503E-01	0.	7.607115E-02	4.012830E-03	0.
2	9.0000	-4.242643E+00	-7.645503E-01	-1.019141E-12	7.607115E-02	4.012830E-03	4.412756E-15
3	18.0000	-4.242643E+00	-7.645503E-01	-9.899566E-14	7.607115E-02	4.012830E-03	2.754324E-15
4	27.0000	-4.242643E+00	-7.645503E-01	-1.248250E-13	7.607115E-02	4.012830E-03	5.646516E-16
5	36.0000	-4.242643E+00	-7.645503E-01	8.355089E-13	7.607115E-02	4.012830E-03	-3.940906E-15
6	45.0000	-4.242643E+00	-7.645503E-01	0.	7.607115E-02	4.012830E-03	0.

Fig. 7-18 (Cont.)

```
ROW  11.  X=    22.4467
COL        Y           MX              MY              MXY              MX              MY              MXY
  1     0.0000    -4.589472E+00   -3.091624E+00   0.               -1.907623E-01   -1.639607E-02   0.
  2     9.0000    -4.589472E+00   -3.091624E+00   -4.494350E-13    -1.907623E-01   -1.639607E-02   -1.753281E-15
  3    18.0000    -4.589472E+00   -3.091624E+00   -3.450202E-13    -1.907623E-01   -1.639607E-02   -1.177145E-15
  4    27.0000    -4.589472E+00   -3.091624E+00    2.957512E-13    -1.907623E-01   -1.639607E-02   -1.758872E-16
  5    36.0000    -4.589472E+00   -3.091624E+00    2.466331E-13    -1.907623E-01   -1.639607E-02    6.979526E-16
  6    45.0000    -4.589472E+00   -3.091624E+00   0.               -1.907623E-01   -1.639607E-02   0.

ROW  15.  X=    33.6670
COL        Y           MX              MY              MXY              MX              MY              MXY
  1     0.0000    -5.431668E+00   -5.431668E-01   0.                5.309384E-01    5.361313E-02   0.
  2     9.0000    -5.431668E+00   -5.431668E-01   -7.281490E-15     5.309384E-01    5.361313E-02   -2.810149E-18
  3    18.0000    -5.431668E+00   -5.431668E-01   -6.231940E-14     5.309384E-01    5.361313E-02   -2.431817E-17
  4    27.0000    -5.431668E+00   -5.431668E-01    1.199586E-13     5.309384E-01    5.361313E-02    4.680997E-17
  5    36.0000    -5.431668E+00   -5.431668E-01   -3.645900E-14     5.309384E-01    5.361313E-02   -1.422695E-17
  6    45.0000    -5.431668E+00   -5.431668E-01   0.                5.309384E-01    5.361313E-02   0.

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CFBS
    CCC(I,1)           CCC(I,2)           CCC(I,3)           CCC(I,4)           CCC(I,5)           CCC(I,6)

    2.040816E+06       0.                 0.                 0.                 0.                 0.
    2.040816E+05       1.020408E+06       0.                 0.                 0.                 0.
    0.                 0.                 4.000000E+05       0.                 0.                 0.
    0.                 0.                 0.                 6.802721E+03       0.                 0.
    0.                 0.                 0.                 6.802721E+02       3.401361E+03       0.
    0.                 0.                 0.                 0.                 0.                 1.333333E+03
 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=   17.894.  NR OF IO REQUESTS (TAPE2)=   106.     WORDS USED (TAPE2)=  83508.    WORDS TRANSFERRED (TAPE2)=     490458


DETERMINANT OF STIFFNESS MATRIX=   3.2638477E+06*10.**   668.   NUMBER OF NEGATIVE ROOTS =   0
      95 NODES.        432 EQUATIONS.   MAXIMUM BAND WIDTH =          54
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=   18.064.  NR OF IO REQUESTS (TAPE2)=   112.     WORDS USED (TAPE2)=  83508.    WORDS TRANSFERRED (TAPE2)=     519318

EIGENVALUE SHIFT=  -0.          ,     NUMBER OF NEGATIVE   ROOTS=      0

ITERATION        EIGENVALUE                  EIGENVALUE
             (RAYLEIGH QUOTIENT)        (ACCELERATED ESTIMATE)
    0          1.9071763E+03               7.2759576E-12
    1          8.5542560E+03              -8.5415332E+02
    2          6.9551846E+03               7.2712709E+03
    3          6.5570065E+03               6.4260771E+03
    4          6.4437034E+03               6.3986372E+03
    5          6.3781409E+03               6.2881073E+03
    6          6.2968993E+03               6.7178480E+03
    7          6.1784789E+03               6.5556715E+03
    8          6.0127512E+03               6.5933269E+03
    9          5.8009277E+03               6.7743175E+03
   10          5.5617729E+03               7.6544232E+03
   11          5.3293309E+03              -4.2186121E+03
   12          5.1313500E+03               4.8671615E+03
   13          4.9846587E+03               4.5567704E+03
   14          4.8852135E+03               4.6758989E+03
   15          4.8219992E+03               4.7117843E+03

 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=   27.582.  NR OF IO REQUESTS (TAPE2)=   247.     WORDS USED (TAPE2)=  98458.    WORDS TRANSFERRED (TAPE2)=    1298952
```

Fig. 7-18 (Cont.)

```
DETERMINANT OF STIFFNESS MATRIX=   2.1776530E+04*10.**   660.   NUMBER CF NEGATIVE ROOTS =   0
      96 NODES.        432 EQUATIONS.   MAXIMUM BAND WIDTH =         54
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=   20.580.  NR OF IO REQUESTS (TAPE2)=   253.     WORDS USED (TAPE2)=   98450.     WORDS TRANSFERRED (TAPE2)=     1319812

EIGENVALUE SHIFT=   3.3753994E+03,     NUMBER OF NEGATIVE   ROOTS=       0

ITERATION          EIGENVALUE                        EIGENVALUE
               (RAYLEIGH QUOTIENT)              (ACCELERATED ESTIMATE)
    0           4.7709116E+03                       4.5556913E+03
    1           4.7416049E+03                       4.7021718E+03
    2           4.7366926E+03                       4.7357037E+03
    3           4.7372215E+03                       4.7371702E+03
    4           4.7385083E+03                       4.7363243E+03
    5           4.7395520E+03                       4.7440285E+03
    6           4.7402512E+03                       4.7416635E+03
    7           4.7406861E+03                       4.7414038E+03
    8           4.7409482E+03                       4.7413453E+03

 THE BUCKLING LOAD BASED ON LINEAR BIFURCATION THEORY IS  4.741345E+03 TIMES THE STARTING LOAD.

  ROW   1,  X=     0.0000
  COL       Y             W                  V              U              BETAX           BETAY
    1    0.0000    0.                 1.084202E-19   0.             0.              -1.084202E-20
    2    9.0000    0.                 2.168404E-19   5.421011E-20   -2.168404E-20   -2.168404E-20
    3   18.0000    0.                 0.             1.084202E-19   -4.336809E-20    0.
    4   27.0000    0.                 5.421011E-20   1.084202E-19   -4.336809E-20   -5.421011E-21
    5   36.0000    0.                 0.             1.084202E-19   -4.336809E-20    0.
    6   45.0000    0.                 0.             1.084202E-19   -4.336809E-20    0.

  ROW   6,  X=    11.2223
  COL       Y             W                  V              U              BETAX           BETAY
    1    0.0000    0.                 6.074107E-03   0.             0.               9.283555E-02
    2    9.0000    1.362912E-01       5.776152E-03   -5.160641E-02  -8.416573E-02    8.810709E-02
    3   18.0000    2.507015E-01       4.911999E-03   -9.801907E-02  -1.590601E-01    7.453650E-02
    4   27.0000    3.551532E-01       3.567436E-03   -1.346697E-01  -2.196575E-01    5.378373E-02
    5   36.0000    4.166326E-01       1.074940E-03   -1.500836E-01  -2.578598E-01    2.811811E-02
    6   45.0000    4.377210E-01       0.             -1.661265E-01  -2.709840E-01    0.

  ROW  11,  X=    22.4447
  COL       Y             W                  V              U              BETAX           BETAY
    1    0.0000    0.                 2.684425E-03   0.             0.              -2.577030E-01
    2    9.0000    -3.118542E-01      2.571939E-03   -2.966129E-02  6.627672E-04    -2.444743E-01
    3   18.0000    -5.916073E-01      2.229941E-03   -5.635475E-02  1.290999E-03    -2.065095E-01
    4   27.0000    -8.110420E-01      1.650018E-03   -7.745610E-02  1.853572E-03    -1.480617E-01
    5   36.0000    -9.519530E-01      8.077902E-04   -9.095087E-02  2.241040E-03    -7.773549E-02
    6   45.0000    -9.999694E-01      0.             -9.550953E-02  2.302550E-03     0.

  ROW  16,  X=    33.6670
  COL       Y             W                  V              U              BETAX           BETAY
    1    0.0000    0.                 0.             0.             0.               0.
    2    9.0000    0.                 8.673617E-19   -8.673617E-19  6.957531E-20    -5.076900E-20
    3   18.0000    0.                 1.734723E-18   -8.673617E-19  6.957531E-20    -1.015380E-19
    4   27.0000    0.                 4.336809E-19   -1.734723E-18  1.391506E-19    -2.538450E-20
    5   36.0000    0.                 1.084202E-19   0.             0.              -6.346124E-21
    6   45.0000    0.                 0.             -1.734723E-18  1.391506E-19     0.
CP SECONDS=   29.995.  NR OF IO REQUESTS (TAPE2)=   305.     WORDS USED (TAPE2)=   98450.     WORDS TRANSFERRED (TAPE2)=     1640933
```

Fig. 7-18 (Cont.)

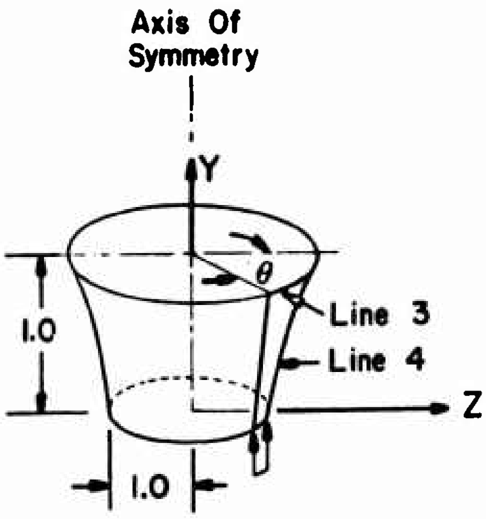
7.7 SAMPLE CASE 7 – HYPERBOLOID

The critical axial line load according to bifurcation theory is to be determined for a hyperboloid shell segment clamped at the large circular edge and constraint in all but the axial direction at the other edge. The geometry of the shell is shown in Fig. 7-22.

Analysis with BOSOR4 (Ref. 15) shows that for this shell the critical axial line load is 1740 lb/in. for a mode with 6 circumferential waves. Therefore, a shell covering 1/24 of the circumference can be analyzed.

The meridional boundary conditions are the same as for Sample Case 4. The input cards associated with this case are displayed in Fig. 7-23. Portions of the output are presented in Fig. 7-24.

The critical axial line load determined here is 1927 lb/in. The discrepancy in the critical load is due to the coarseness of the grid in the longitudinal direction, selected here to reduce the run time.



The Equation Of The Generator Is:

$$\frac{Z^2}{R^2} - \frac{Y^2}{R_1^2} = 1$$

And $R = R_1 = 1.0$

$\theta = 15°$

Fig. 7-19 Sample Case 7 – Hyperboloid

SAMPLE CASE 7
INPUT

```
    SAMPLE CASE 7 - HYPERBOLOID                                                           C-1
      9     1     1                                                                       G-1
0.0           35.267     15.0         1.0          1.0                                    G-2
      9     9                                                                             D-1
      0     4     2     4                                                                 B-1
      0     0     1     0                                                                 B-2
      1     0 1.                                                                          L-1
0.          0.           1.               0     0     2     1     0                       L-2
 0.0          1700.0           1     1    20                                              P-1A
      0     4     2     5                                                                 P-1A1
      0     0     1     0                                                                 P-1A2
      0     0     1     4     0     1     1                                               O-1
0.          17.          25.          35.                                                 O-2
      2                                                                                   M-1
0.025       10000000. 0.3             0.0                                                 M-2B
```

Fig. 7-20 Display of Input Cards for Sample Case 7

```
   SAMPLE CASE 7 - HYPERBOLOID
BUCKLING ANALYSIS.           1 LOAD PATTERNS.

TYPE OF SURFACE IS HYPERBOLOID

SURFACE CONSTANTS =  0.            , 3.5267000E+01, 1.5000000E+01, 1.0000000E+00, 1.0000000E+00,

 BLANK COMMON ARRAY WORKING SPACE=     15000

  FINITE DIFFERENCE MESH.   9 ROWS,    9 COLUMNS.  MESH SPACING, H=   4.4084, K=   1.8750

NRW1=  -0, NRW2= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS SET BY IFREE . IFREE = 0, 0, 1, 0,
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS   CLAMPED
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD   DATA

CARD COUNT =     1

 USER-LOAD FLAG =     0.  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP = -0.            MAXIMUM LOAD =  1.0000000E+00

      PZ            PY               PX          JZ    JY    JX  ROW  COL
 0.            0.              1.0000000E+00      0     0     2    1    0

BOUNDARY CONDITIONS FOR BUCKLING DISPLACEMENTS
BOUNDARY CONDITION AT LINE 1 IS SET BY IFREE . IFREE = 0, 0, 1, 0,
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS   CLAMPED
BOUNDARY CONDITION AT LINE 4 IS  ANTI-METRIC

 ISHIFT   ITERAT   SHIFT
   1        23      1.70000E+03

IPX=     4 IPY=     0 IPRO=     1 IPRS=     1 IPLOT=    -0

IWALL=     2,  NSTRI=   -0,  NRING=   -0,  IP=   -0,  IM=   -0,  JM=   -0

  AT =  2.5000000E-02  EX1 =  1.0000000E+07 XNU =  3.0000000E-01
   Z =  0.             EY1 =  1.0000000E+07   G =  3.8461538E+06

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CFB2
     CCC(I,1)             CCC(I,2)             CCC(I,3)             CCC(I,4)             CCC(I,5)             CCC(I,6)

       2.747253E+05         0.                   0.                   0.                   0.                   0.
       8.241758E+04         2.747253E+05         0.                   0.                   0.                   0.
       0.                   0.                   9.615385E+04         0.                   0.                   0.
       0.                   0.                   0.                   1.430861E+01         0.                   0.
       0.                   0.                   0.                   4.292582E+00         1.430861E+01         0.
       0.                   0.                   0.                   0.                   0.                   5.000013E+00
```

Fig. 7-21 Excerpt of Output Sample Core 7

```
 CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=    1.646.  NR OF IO REQUESTS (TAPE2)=    13.       WORDS USED (TAPE2)=   41910.     WORDS TRANSFERRED (TAPE2)=        43950

 FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=    5.975.  NR OF IO REQUESTS (TAPE2)=    26.       WORDS USED (TAPE2)=   62689.     WORDS TRANSFERRED (TAPE2)=        94133

 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=    6.345.  NR OF IO REQUESTS (TAPE2)=    32.       WORDS USED (TAPE2)=   76579.     WORDS TRANSFERRED (TAPE2)=       128802

DISPLACEMENT COMPONENT V      X= .  3.3003E+01       Y=    1.4062E+01,   HAS BEEN ELIMINATED
  THE INITIAL MAIN DIAGONAL VALUE =  1.2408583E+06,   THE FINAL VALUE =  1.0952353E-06

DETERMINANT OF STIFFNESS MATRIX=   4.6778490E+00*10.**   510.   NUMBER OF NEGATIVE ROOTS =   0
       81 NODES.        363 EQUATIONS.    MAXIMUM BAND WIDTH =          72
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=    7.516.  NR OF IO REQUESTS (TAPE2)=    38.       WORDS USED (TAPE2)=   76579.     WORDS TRANSFERRED (TAPE2)=       157766


LINEAR SOLUTION.  PA=   1.000000E+00.    PB=   0.
ROW  8.   X=     30.859, AXIAL LOAD=      -3.841068E-01,BENDING MOMENT=      0.

  ROW   1.  X=     0.0000
                        OUTER SURFACE                                   INNER SURFACE
    X         Y        SIGMAX       SIGMAY          TAU          SIGMAX       SIGMAY           TAU    SIGMA(STRNGR)  SIGMA(RING)
  0.000     0.000    9.0548E+00   3.1604E+00   0.                -8.8522E+01  -2.7009E+01   0.
  0.000     1.875    9.0548E+00   3.1604E+00   2.2340E-12   -8.8522E+01  -2.7009E+01   2.6132E-12
  0.000     3.750    9.0548E+00   3.1604E+00   1.3078E-12   -8.8522E+01  -2.7009E+01   1.7314E-12
  0.000     5.625    9.0549E+00   3.1604E+00  -5.4666E-12   -8.8522E+01  -2.7009E+01  -5.6082E-12
  0.000     7.500    9.0548E+00   3.1604E+00  -1.2671E-12   -8.8522E+01  -2.7009E+01  -3.9167E-13
  0.000     9.375    9.0548E+00   3.1604E+00  -1.9445E-12   -8.8522E+01  -2.7009E+01  -2.0124E-12
  0.000    11.250    9.0549E+00   3.1604E+00  -7.3951E-12   -8.8522E+01  -2.7009E+01  -7.0343E-12
  0.000    13.125    9.0548E+00   3.1604E+00  -1.6618E-13   -8.8522E+01  -2.7009E+01  -4.0353E-13
  0.000    15.000    9.0548E+00   3.1604E+00   0.                -8.8522E+01  -2.7009E+01   0.

  ROW   9.  X=    35.2670
                        OUTER SURFACE                                   INNER SURFACE
    X         Y        SIGMAX       SIGMAY          TAU          SIGMAX       SIGMAY           TAU    SIGMA(STRNGR)  SIGMA(RING)
 35.267     0.000   -2.9616E+01  -8.6642E+00   0.                -3.7604E+01  -1.1502E+01   0.
 35.267     1.875   -2.9616E+01  -8.6642E+00   2.1039E-12   -3.7604E+01  -1.1502E+01   2.0440E-12
 35.267     3.750   -2.9616E+01  -8.6642E+00   3.4965E-12   -3.7604E+01  -1.1502E+01   3.3990E-12
 35.267     5.625   -2.9616E+01  -8.6642E+00   5.5273E-12   -3.7604E+01  -1.1502E+01   5.3700E-12
 35.267     7.500   -2.9616E+01  -8.6642E+00   4.5332E-12   -3.7604E+01  -1.1502E+01   4.4043E-12
 35.267     9.375   -2.9616E+01  -8.6642E+00   6.6516E-12   -3.7604E+01  -1.1502E+01   6.4624E-12
 35.267    11.250   -2.9616E+01  -8.6642E+00   3.1629E-12   -3.7604E+01  -1.1502E+01   3.0725E-12
 35.267    13.125   -2.9616E+01  -8.6642E+00   4.1366E-12   -3.7604E+01  -1.1502E+01   4.0189E-12
 35.267    15.000   -2.9616E+01  -8.6642E+00   0.                -3.7604E+01  -1.1502E+01   0.
CP SECONDS=    8.631.  NR OF IO REQUESTS (TAPE2)=    63.       WORDS USED (TAPE2)=   77891.     WORDS TRANSFERRED (TAPE2)=       264509

  ROW   1.  X=     0.0000
  COL        Y              W                  V                     U               BETAX            BETAY
    1     0.0000      0.               0.                    4.759533E-06    4.759533E-06    0.
    2     1.8750      0.               0.                    4.759533E-06    4.759533E-06    0.
    3     3.7500      0.               0.                    4.759533E-06    4.759533E-06    0.
    4     5.6250      0.               0.                    4.759533E-06    4.759533E-06    0.
    5     7.5000      0.              -3.851068E-34          4.759533E-06    4.759533E-06    3.851068E-34
    6     9.3750      0.               0.                    4.759533E-06    4.759533E-06    0.
    7    11.2500      0.              -1.925930E-34          4.759533E-06    4.759533E-06    1.925930E-34
    8    13.1250      0.               0.                    4.759533E-06    4.759533E-06    0.
    9    15.0000      0.               0.                    4.759533E-06    4.759533E-06    0.
```

Fig. 7-21 (Cont.)

ROW	5.	X=	17.6335			
COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-3.300858E-06	0.	3.033866E-06	2.405247E-06	0.
2	1.8750	-3.300858E-06	2.212324E-20	3.033866E-06	2.405247E-06	7.436020E-18
3	3.7500	-3.300858E-06	3.498096E-20	3.033866E-06	2.405247E-06	1.030916E-17
4	5.6250	-3.300858E-06	3.994749E-20	3.033866E-06	2.405247E-06	8.547471E-18
5	7.5000	-3.300858E-06	3.913289E-20	3.033866E-06	2.405247E-06	6.016026E-18
6	9.3750	-3.300858E-06	3.305291E-20	3.033866E-06	2.405247E-06	3.574451E-18
7	11.2500	-3.300858E-06	2.078505E-20	3.033866E-06	2.405247E-06	1.934365E-18
8	13.1250	-3.300858E-06	1.277992E-20	3.033866E-06	2.405247E-06	-2.171422E-19
9	15.0000	-3.300858E-06	0.	3.033866E-06	2.405247E-06	0.

ROW	7.	X=	26.4502			
COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-2.415731E-06	0.	1.765594E-06	5.222612E-06	0.
2	1.8750	-2.415731E-06	2.630067E-20	1.765594E-06	5.222612E-06	1.022300E-18
3	3.7500	-2.415731E-06	5.400350E-20	1.765594E-06	5.222612E-06	5.715720E-18
4	5.6250	-2.415731E-06	8.140366E-20	1.765594E-06	5.222612E-06	1.236029E-17
5	7.5000	-2.415731E-06	1.010554E-19	1.765594E-06	5.222612E-06	1.863406E-17
6	9.3750	-2.415731E-06	1.073296E-19	1.765594E-06	5.222612E-06	2.263872E-17
7	11.2500	-2.415731E-06	9.262643E-20	1.765594E-06	5.222612E-06	2.242971E-17
8	13.1250	-2.415731E-06	5.805613E-20	1.765594E-06	5.222612E-06	1.332775E-17
9	15.0000	-2.415731E-06	0.	1.765594E-06	5.222612E-06	0.

ROW	9.	X=	35.2670			
COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	-8.470329E-22	-1.629486E-22	0.
2	1.8750	0.	0.	-8.470329E-22	-1.629486E-22	0.
3	3.7500	0.	0.	-1.694066E-21	-3.258972E-22	0.
4	5.6250	0.	0.	-8.470329E-22	-1.629486E-22	0.
5	7.5000	0.	0.	-8.470329E-22	-1.629486E-22	0.
6	9.3750	0.	2.808095E-34	-8.470329E-22	-1.629486E-22	-1.667609E-34
7	11.2500	0.	3.851860E-34	-1.694066E-21	-3.258972E-22	-2.223586E-34
8	13.1250	0.	0.	0.	0.	0.
9	15.0000	0.	0.	0.	0.	0.

ROW	1.	X=	0.0000				
COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-9.933461E-01	-2.980030E-01	0.	5.082147E-03	1.571725E-03	0.
2	1.8750	-9.933461E-01	-2.980030E-01	6.059876E-14	5.082147E-03	1.571725E-03	-1.975101E-17
3	3.7500	-9.933461E-01	-2.980030E-01	3.798024E-14	5.082147E-03	1.571725E-03	-2.210628E-17
4	5.6250	-9.933461E-01	-2.980030E-01	-1.394347E-13	5.082147E-03	1.571725E-03	1.153810E-17
5	7.5000	-9.933461E-01	-2.980030E-01	-2.073514E-14	5.082147E-03	1.571725E-03	-4.559750E-17
6	9.3750	-9.933461E-01	-2.980030E-01	-4.946052E-14	5.082147E-03	1.571725E-03	3.535380E-18
7	11.2500	-9.933461E-01	-2.980030E-01	-1.803675E-13	5.082147E-03	1.571725E-03	-1.078828E-17
8	13.1250	-9.933461E-01	-2.980030E-01	-8.121304E-15	5.082147E-03	1.571725E-03	1.652906E-17
9	15.0000	-9.933461E-01	-2.980030E-01	0.	5.082147E-03	1.571725E-03	0.

ROW	5.	X=	17.6335				
COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-9.891397E-01	-8.245513E-01	0.	-5.062795E-04	-1.259697E-04	0.
2	1.8750	-9.891397E-01	-8.245513E-01	3.908929E-14	-5.062795E-04	-1.259697E-04	-9.133486E-16
3	3.7500	-9.891397E-01	-8.245513E-01	6.872031E-14	-5.062795E-04	-1.259697E-04	-1.644092E-15
4	5.6250	-9.891397E-01	-8.245513E-01	2.371751E-14	-5.062795E-04	-1.259697E-04	-1.930227E-15
5	7.5000	-9.891397E-01	-8.245513E-01	-7.932666E-14	-5.062795E-04	-1.259697E-04	-1.749291E-15
6	9.3750	-9.891397E-01	-8.245513E-01	3.000203E-14	-5.062795E-04	-1.259697E-04	-1.075601E-15
7	11.2500	-9.891397E-01	-8.245513E-01	-6.012242E-14	-5.062795E-04	-1.259697E-04	-3.632187E-16
8	13.1250	-9.891397E-01	-8.245513E-01	4.984312E-14	-5.062795E-04	-1.259697E-04	1.214787E-18
9	15.0000	-9.891397E-01	-8.245513E-01	0.	-5.062795E-04	-1.259697E-04	0.

Fig. 7-21 (Cont.)

```
ROW   7,  X=    26.4502
COL      Y            NX              NY              NXY             MX              MY              MXY
  1    0.0000   -9.651799E-01   -5.880540E-01   0.              -1.912428E-06   5.057224E-06   0.
  2    1.8750   -9.651799E-01   -5.880540E-01   -2.552977E-14   -1.912428E-06   5.057224E-06   6.878122E-16
  3    3.7500   -9.651799E-01   -5.880540E-01   -1.413251E-14   -1.912428E-06   5.057224E-06   1.142988E-15
  4    5.6250   -9.651799E-01   -5.880540E-01    2.368529E-14   -1.912428E-06   5.057224E-06   1.255389E-15
  5    7.5000   -9.651799E-01   -5.880540E-01   -3.382506E-14   -1.912428E-06   5.057224E-06   1.133282E-15
  6    9.3750   -9.651799E-01   -5.880540E-01    1.235951E-14   -1.912428E-06   5.057224E-06   8.473273E-16
  7   11.2500   -9.651799E-01   -5.880540E-01    2.604764E-15   -1.912428E-06   5.057224E-06   4.529200E-16
  8   13.1250   -9.651799E-01   -5.880540E-01    5.127281E-14   -1.912428E-06   5.057224E-06   1.731012E-16
  9   15.0000   -9.651799E-01   -5.880540E-01   0.              -1.912428E-06   5.057224E-06   0.

ROW   9,  X=    35.2670
COL      Y            NX              NY              NXY             MX              MY              MXY
  1    0.0000   -8.402533E-01   -2.520760E-01   0.               4.160289E-04   1.477984E-04   0.
  2    1.8750   -8.402533E-01   -2.520760E-01   5.184839E-14     4.160289E-04   1.477984E-04   3.117794E-18
  3    3.7500   -8.402533E-01   -2.520760E-01   8.621904E-14     4.160289E-04   1.477984E-04   5.184601E-18
  4    5.6250   -8.402533E-01   -2.520760E-01   1.362166E-13     4.160289E-04   1.477984E-04   8.191101E-18
  5    7.5000   -8.402533E-01   -2.520760E-01   1.117189E-13     4.160289E-04   1.477984E-04   6.717983E-18
  6    9.3750   -8.402533E-01   -2.520760E-01   1.639245E-13     4.160289E-04   1.477984E-04   9.857258E-18
  7   11.2500   -8.402533E-01   -2.520760E-01   7.793761E-14     4.160289E-04   1.477984E-04   4.686615E-18
  8   13.1250   -8.402533E-01   -2.520760E-01   1.019443E-13     4.160289E-04   1.477984E-04   6.130208E-18
  9   15.0000   -8.402533E-01   -2.520760E-01   0.               4.160289E-04   1.477984E-04   0.

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CF02
     CCC(I,1)          CCC(I,2)          CCC(I,3)          CCC(I,4)          CCC(I,5)          CCC(I,6)

     2.747253E+05      0.                0.                0.                0.                0.
     8.241758E+04      2.747253E+05      0.                0.                0.                0.
     0.                0.                9.615385E+04      0.                0.                0.
     0.                0.                0.                1.430861E+01      0.                0.
     0.                0.                0.                4.292582E+00      1.430861E+01      0.
     0.                0.                0.                0.                0.                5.008013E+00
 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=   15.389.  NR OF IO REQUESTS (TAPE2)=    98.      WORDS USED (TAPE2)=  77603.     WORDS TRANSFERRED (TAPE2)=      419795


DETERMINANT OF STIFFNESS MATRIX=   7.1132378E+02*10.**    508.   NUMBER OF NEGATIVE ROOTS =   0
      81 NODES.        363 EQUATIONS.    MAXIMUM BAND WIDTH =         72
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=   16.512.  NR OF IO REQUESTS (TAPE2)=   104.      WORDS USED (TAPE2)=  77603.     WORDS TRANSFERRED (TAPE2)=      448759

EIGENVALUE SHIFT=   1.7000000E+03,     NUMBER OF NEGATIVE   ROOTS=        0

ITERATION          EIGENVALUE                     EIGENVALUE
               (RAYLEIGH QUOTIENT)           (ACCELERATED ESTIMATE)
    0            1.7520807E+03                  1.7000000E+03
    1            1.9449135E+03                  1.6807292E+03
    2            1.9291781E+03                  1.9303225E+03
    3            1.9271140E+03                  1.9268194E+03
    4            1.9267213E+03                  1.9266262E+03
    5            1.9266275E+03                  1.9265981E+03

 THE BUCKLING LOAD BASED ON LINEAR BIFURCATION THEORY IS  1.926598E+03 TIMES THE STARTING LOAD.
```

Fig. 7-21 (Cont.)

ROW 1. X= 0.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	2.775558E-17	0.	0.	-2.775558E-17
2	1.8750	0.	2.775558E-17	-2.648773E-02	-2.648773E-02	-2.775558E-17
3	3.7500	0.	2.775558E-17	-5.195755E-02	-5.195755E-02	-2.775558E-17
4	5.6250	0.	4.163336E-17	-7.543067E-02	-7.543067E-02	-4.163336E-17
5	7.5000	0.	1.387779E-17	-9.600504E-02	-9.600504E-02	-1.387779E-17
6	9.3750	0.	1.387779E-17	-1.128900E-01	-1.128900E-01	-1.387779E-17
7	11.2500	0.	6.938894E-18	-1.254366E-01	-1.254366E-01	-6.938894E-18
8	13.1250	0.	1.734723E-18	-1.331628E-01	-1.331628E-01	-1.734723E-18
9	15.0000	0.	0.	-1.357716E-01	-1.357716E-01	0.

ROW 5. X= 17.6335

COL	Y	W	V	U	DETAX	BETAY
1	0.0000	0.	1.420000E-01	0.	0.	5.525769E+00
2	1.8750	1.951504E-01	1.392715E-01	-6.568456E-04	1.161697E-02	5.419593E+00
3	3.7500	3.828012E-01	1.311909E-01	-1.288449E-03	2.278750E-02	5.105145E+00
4	5.6250	5.557413E-01	1.180687E-01	-1.870538E-03	3.308231E-02	4.594509E+00
5	7.5000	7.073244E-01	1.004091E-01	-2.380743E-03	4.210579E-02	3.907308E+00
6	9.3750	8.317255E-01	7.889095E-02	-2.799458E-03	4.951115E-02	3.069952E+00
7	11.2500	9.241639E-01	5.434103E-02	-3.110591E-03	5.501384E-02	2.114620E+00
8	13.1250	9.810172E-01	2.770282E-02	-3.302186E-03	5.840237E-02	1.078024E+00
9	15.0000	1.000308E+00	0.	-3.366879E-03	5.954654E-02	0.

ROW 7. X= 26.4502

COL	Y	W	V	U	BETAX	DCTAY
1	0.0000	0.	4.601916E-02	0.	0.	1.404532E+00
2	1.8750	5.433397E-02	4.513491E-02	1.480385E-02	-5.826534E-01	1.377544E+00
3	3.7500	1.065799E-01	4.251616E-02	2.746796E-02	-1.142916E+00	1.297618E+00
4	5.6250	1.547301E-01	3.826353E-02	3.987730E-02	-1.659256E+00	1.167825E+00
5	7.5000	1.969340E-01	3.254046E-02	5.075418E-02	-2.111833E+00	9.931539E-01
6	9.3750	2.315699E-01	2.556687E-02	5.968060E-02	-2.483253E+00	7.803100E-01
7	11.2500	2.573067E-01	1.761077E-02	6.631353E-02	-2.769242E+00	5.374910E-01
8	13.1250	2.731553E-01	8.977892E-03	7.039807E-02	-2.929196E+00	2.740105E-01
9	15.0000	2.785067E-01	0.	7.177725E-02	-2.986582E+00	0.

ROW 9. X= 35.2670

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	5.551115E-17	0.	0.	-3.204525E-17
2	1.8750	0.	2.775558E-17	-1.387779E-17	-2.669750E-18	-1.602262E-17
3	3.7500	0.	5.551115E-17	-1.387779E-17	-2.669750E-18	-3.204525E-17
4	5.6250	0.	2.775558E-17	-1.387779E-17	-2.669750E-18	-1.602262E-17
5	7.5000	0.	2.775558E-17	-2.775558E-17	-5.339499E-18	-1.602262E-17
6	9.3750	0.	2.775558E-17	-2.775558E-17	-5.339499E-18	-1.602262E-17
7	11.2500	0.	0.	-5.551115E-17	-1.067900E-17	0.
8	13.1250	0.	6.938894E-18	0.	0.	-4.005656E-18
9	15.0000	0.	0.	0.	0.	0.

CP SECONDS= 18.664. NR OF IO REQUESTS (TAPE2)= 163. WORDS USED (TAPE2)= 92085. WORDS TRANSFERRED (TAPE2)= 760787

Fig. 7-21 (Cont.)

7.8 SAMPLE CASE 8 – PARABOLOID

The critical external pressure according to bifurcation theory is to be determined for a paraboloid shell with clamped circular edges. The geometry of the shell is shown in Fig. 7-22.

Analysis with BOSOR4 (Ref. 15) shows that for this shell, the critical external pressure is 714 psi for a mode with 10 circumferential waves. Therefore, a shell covering 1/40 of the circumference can be analyzed.

The meridional boundary conditions are the same as for Sample Case 4. The input cards associated with this case are displayed in Fig. 7-23. Portions of the output are presented in Fig. 7-24.

The critical external pressure determined here is 703 psi.

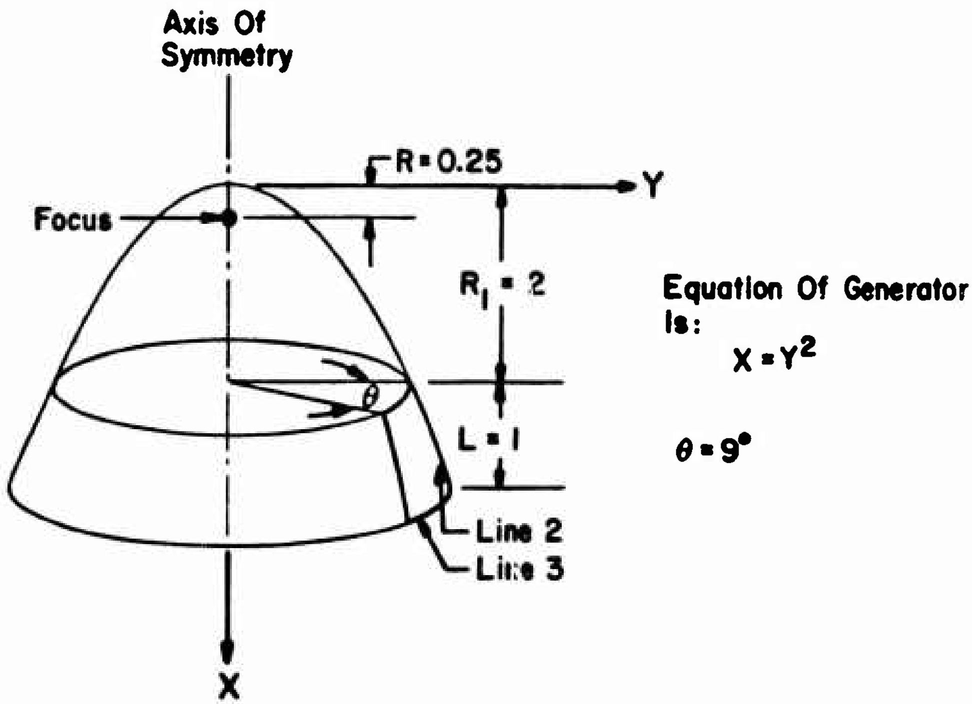


Fig. 7-22 Sample Case 8 – Paraboloid

SAMPLE CASE 8
INPUT

```
     SAMPLE CASE 8 - PARABOLOID                                  C-1
     5    1    1                                                 G-1
 1.         9.         .25        2.                             G-2
   11     7                                                      D-1
     2    4    2    4                                            B-1
     1    0 1.         1.         1.                             L-1
-1.         0.         0.           5    0    0    0    0        L-2
                            1    2   20                          P-1A
     2    4    2    5                                            P-1A1
     0    0    1    4    0    1    1                             O-1
0.          .5         .8         1.                             O-2
     2                                                           M-1
 .025       10000000. .3          0.                             M-2B
```

Fig. 7-23 Display of Input Cards for Sample Case 8

```
    SAMPLE CASE 8 - PARABOLOID
BUCKLING ANALYSIS.            1 LOAD PATTERNS.

TYPE OF SURFACE IS PARABOLOID

SURFACE CONSTANTS =  1.0000000E+00, 9.0000000E+00, 2.5000000E-01, 2.0000000E+00,

 BLANK COMMON ARRAY WORKING SPACE=     15000

  FINITE DIFFERENCE MESH.  11 ROWS,     7 COLUMNS.  MESH SPACING. H=    .1000, K=   1.5000

NRW1=  -0, NRW2= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS    CLAMPED
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS    CLAMPED
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =      1

 USER-LOAD FLAG =      0,  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP =  1.0000000E+00,  MAXIMUM LOAD =  1.0000000E+00

       PZ              PY               PX           JZ    JY    JX   ROW   COL
-1.0000000E+00 0.                  0.                5     0     0     0     0

BOUNDARY CONDITIONS FOR BUCKLING DISPLACEMENTS
BOUNDARY CONDITION AT LINE 1 IS    CLAMPED
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS    CLAMPED
BOUNDARY CONDITION AT LINE 4 IS  ANTI-METRIC

  ISHIFT   ITERAT   SHIFT
   2         20     -0.

IPX=     4 IPY=     0 IPRD=     1 IPRS=     1 IPLOT=   -0

IWALL=     2.  ASTRI=    -0,  NRING=   -0,  IP=   -0,  IM=   -0,  JM=   -0

   AT =  2.5000000E-02  EX1 =  1.0000000E+07 XNU =  3.0000000E-01
    Z =  0.             EY1 =  1.0000000E+07   G =  3.8461538E+06


 THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CFB2
      CCC(I,1)             CCC(I,2)             CCC(I,3)             CCC(I,4)             CCC(I,5)             CCC(I,6)

                                                                                                               0.
                                                                     0.                   0.                   0.
                                                0.                   0.                   0.                   0.
     2.747253E+05          0.                   0.                   0.                   0.                   0.
     8.241758E+04          2.747253E+05         9.615385E+04         1.430861E+01         1.430861E+01         0.
     0.                    0.                   0.                   4.292582E+00         0.                   5.008013E+00
     0.                    0.                   0.                   0.
     0.                    0.                   0.
     0.                    0.
```

Fig. 7-24 Excerpt of Output for Sample Case 8

```
CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=     1.379.  NR OF IO REQUESTS (TAPE2)=     13.        WORDS USED (TAPE2)=   41922.     WORDS TRANSFERRED (TAPE2)=       43970

FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=     5.034.  NR OF IO REQUESTS (TAPE2)=     26.        WORDS USED (TAPE2)=   62627.     WORDS TRANSFERRED (TAPE2)=       94159

ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=     5.389.  NR OF IO REQUESTS (TAPE2)=     32.        WORDS USED (TAPE2)=   76601.     WORDS TRANSFERRED (TAPE2)=      128838

DISPLACEMENT COMPONENT V       X=    9.5000E-01      Y=    8.2500E+00,   HAS BEEN ELIMINATED
  THE INITIAL MAIN DIAGONAL VALUE =  5.1273012E+05,   THE FINAL VALUE =  8.1211329E-07

DETERMINANT OF STIFFNESS MATRIX=   3.4664726E+01*10.**   470.   NUMBER OF NEGATIVE ROOTS =   0
        77 NODES.         351 EQUATIONS.    MAXIMUM BAND WIDTH =         60
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=     6.293.  NR OF IO REQUESTS (TAPE2)=     38.        WORDS USED (TAPE2)=   76601.     WORDS TRANSFERRED (TAPE2)=      157810


LINEAR SOLUTION.  PA=   1.000000E+00,    PB=   0.
ROW 10,  X=       .930, AXIAL LOAD=       -1.425870E-01,BENDING MOMENT=      0.

  ROW   1,  X=     0.0300
```

		OUTER SURFACE			INNER SURFACE				
X	Y	SIGMAX	SIGMAY	TAU	SIGMAX	SIGMAY	TAU	SIGMA(STRNGR)	SIGMA(RING)
0.000	0.000	6.5975E+01	1.9908E+01	0.	-9.6397E+01	-2.9035E+01	0.		
0.000	1.500	6.5975E+01	1.9908E+01	8.8883E-13	-9.6397E+01	-2.9035E+01	8.5959E-13		
0.000	3.000	6.5975E+01	1.9908E+01	2.9398E-12	-9.6397E+01	-2.9035E+01	2.8434E-12		
0.000	4.500	6.5975E+01	1.9908E+01	1.6415E-12	-9.6397E+01	-2.9035E+01	1.5876E-12		
0.000	6.000	6.5975E+01	1.9908E+01	2.4221E-12	-9.6397E+01	-2.9035E+01	2.3427E-12		
0.000	7.500	6.5975E+01	1.9908E+01	4.2335E-13	-9.6397E+01	-2.9035E+01	4.0947E-13		
0.000	9.000	6.5975E+01	1.9908E+01	0.	-9.6397E+01	-2.9035E+01	0.		

```
  ROW   2,  X=      .1000
```

		OUTER SURFACE			INNER SURFACE				
X	Y	SIGMAX	SIGMAY	TAU	SIGMAX	SIGMAY	TAU	SIGMA(STRNGR)	SIGMA(RING)
.100	0.000	-1.3127E+01	-2.4883E+01	0.	-1.7268E+01	-2.8046E+01	0.		
.100	1.500	-1.3127E+01	-2.4883E+01	-3.5171E-12	-1.7268E+01	-2.8046E+01	5.3028E-12		
.100	3.000	-1.3127E+01	-2.4883E+01	-1.8036E-11	-1.7268E+01	-2.8046E+01	1.2820E-11		
.100	4.500	-1.3127E+01	-2.4883E+01	-1.7808E-11	-1.7268E+01	-2.8046E+01	2.2519E-11		
.100	6.000	-1.3127E+01	-2.4883E+01	-2.0917E-11	-1.7268E+01	-2.8046E+01	2.5930E-11		
.100	7.500	-1.3127E+01	-2.4883E+01	-1.8173E-11	-1.7268E+01	-2.8046E+01	2.1870E-11		
.100	9.000	-1.3127E+01	-2.4883E+01	0.	-1.7268E+01	-2.8046E+01	0.		

```
  ROW   6,  X=      .5000
```

		OUTER SURFACE			INNER SURFACE				
X	Y	SIGMAX	SIGMAY	TAU	SIGMAX	SIGMAY	TAU	SIGMA(STRNGR)	SIGMA(RING)
.500	0.000	-2.4003E+01	-7.0201E+01	0.	-1.3116E+01	-6.7046E+01	0.		
.500	1.500	-2.4003E+01	-7.0201E+01	-1.2798E-12	-1.3116E+01	-6.7046E+01	1.2363E-12		
.500	3.000	-2.4003E+01	-7.0201E+01	2.9887E-12	-1.3116E+01	-6.7046E+01	-4.7291E-12		
.500	4.500	-2.4003E+01	-7.0201E+01	1.0351E-11	-1.3116E+01	-6.7046E+01	-1.1442E-11		
.500	6.000	-2.4003E+01	-7.0201E+01	1.4173E-11	-1.3116E+01	-6.7046E+01	-1.7015E-11		
.500	7.500	-2.4003E+01	-7.0201E+01	1.1164E-11	-1.3115E+01	-6.7046E+01	-1.2580E-11		
.500	9.000	-2.4003E+01	-7.0201E+01	0.	-1.3116E+01	-6.7046E+01	0.		

Fig. 7-24 (Cont.)

```
ROW   9.  X=     .8000
                        OUTER SURFACE                                      INNER SURFACE
   X        Y       SIGMAX       SIGMAY          TAU         SIGMAX       SIGMAY          TAU    SIGMA(STRNGR)   SIGMA(RING)
  .800    0.000  -4.2499E+01  -5.6180E+01   0.             4.1460E-01  -4.2119E+01   0.
  .800    1.500  -4.2499E+01  -5.6180E+01   1.0409E-11     4.1460E-01  -4.2119E+01  -1.3170E-11
  .800    3.000  -4.2499E+01  -5.6180E+01   1.6978E-11     4.1460E-01  -4.2119E+01  -1.9377E-11
  .800    4.500  -4.2499E+01  -5.6180E+01   1.2634E-11     4.1460E-01  -4.2119E+01  -1.7602E-11
  .800    6.000  -4.2499E+01  -5.6180E+01   7.3569E-12     4.1460E-01  -4.2119E+01  -9.9702E-12
  .800    7.500  -4.2499E+01  -5.6180E+01   5.8956E-12     4.1460E-01  -4.2119E+01  -8.2019E-12
  .800    9.000  -4.2499E+01  -5.6180E+01   0.             4.1460E-01  -4.2119E+01   0.

ROW  10.  X=     .9000
                        OUTER SURFACE                                      INNER SURFACE
   X        Y       SIGMAX       SIGMAY          TAU         SIGMAX       SIGMAY          TAU    SIGMA(STRNGR)   SIGMA(RING)
  .900    0.000  -1.6986E+01  -2.5731E+01   0.            -2.5657E+01  -2.7023E+01   0.
  .900    1.500  -1.6986E+01  -2.5731E+01   3.2302E-13    -2.5657E+01  -2.7023E+01   1.6752E-12
  .900    3.000  -1.6986E+01  -2.5731E+01  -2.3394E-12    -2.5657E+01  -2.7023E+01   5.2487E-12
  .900    4.500  -1.6986E+01  -2.5731E+01  -5.6776E-12    -2.5657E+01  -2.7023E+01   9.0076E-12
  .900    6.000  -1.6986E+01  -2.5731E+01  -6.9799E-12    -2.5657E+01  -2.7023E+01   9.5631E-12
  .900    7.500  -1.6986E+01  -2.5731E+01   9.7862E-13    -2.5657E+01  -2.7023E+01   6.3743E-13
  .900    9.000  -1.6986E+01  -2.5731E+01   0.            -2.5657E+01  -2.7023E+01   0.

ROW  11.  X=    1.0000
                        OUTER SURFACE                                      INNER SURFACE
   X        Y       SIGMAX       SIGMAY          TAU         SIGMAX       SIGMAY          TAU    SIGMA(STRNGR)   SIGMA(RING)
 1.000    0.000   7.0025E+01   2.1141E+01   0.            -1.1231E+02  -3.3025E+01   0.
 1.000    1.500   7.0025E+01   2.1141E+01   1.4630E-12    -1.1231E+02  -3.3025E+01   1.4237E-12
 1.000    3.000   7.0025E+01   2.1141E+01   3.1655E-12    -1.1231E+02  -3.3025E+01   3.0789E-12
 1.000    4.500   7.0025E+01   2.1141E+01   2.2230E-12    -1.1231E+02  -3.3025E+01   2.1629E-12
 1.000    6.000   7.0025E+01   2.1141E+01   2.6356E-12    -1.1231E+02  -3.3025E+01   2.5635E-12
 1.000    7.500   7.0025E+01   2.1141E+01   1.1404E-12    -1.1231E+02  -3.3025E+01   1.1092E-12
 1.000    9.000   7.0025E+01   2.1141E+01   0.            -1.1231E+02  -3.3025E+01   0.
CP SECONDS=    7.320.  NR OF IO REQUESTS (TAPE2)=    63.     WORDS USED (TAPE2)=   77113.     WORDS TRANSFERRED (TAPE2)=     264663

ROW   1.  X=    0.0000
COL      Y             W                  V                 U              BETAX            BETAY
  1    0.0000    0.                 0.               0.                0.               0.
  2    1.5000    0.                 9.629650E-35    -1.050791E-22     7.042898E-24    -6.419766E-35
  3    3.0000    0.                 2.407412E-35    -1.050791E-22     7.042898E-24    -1.604942E-35
  4    4.5000    0.                 4.814825E-35    -1.050791E-22     7.042898E-24    -3.209883E-35
  5    6.0000    0.                 0.              -2.117552E-22     1.568580E-23     0.
  6    7.5000    0.                 2.407412E-35    -1.050791E-22     7.042898E-24    -1.604942E-35
  7    9.0000    0.                 0.              -1.050791E-22     7.042898E-24     0.

ROW   6.  X=     .5000
COL      Y             W                  V                 U              BETAX            BETAY
  1    0.0000   -1.049105E-05    0.                 1.009749E-07    -2.978771E-06     0.
  2    1.5000   -1.049105E-05    9.043640E-20       1.009749E-07    -2.978771E-06     3.011107E-17
  3    3.0000   -1.049105E-05    1.701902E-19       1.009749E-07    -2.978771E-06     4.707420E-17
  4    4.5000   -1.049105E-05    1.930886E-19       1.009749E-07    -2.978771E-06     4.651102E-17
  5    6.0000   -1.049105E-05    1.631950E-19       1.009749E-07    -2.978771E-06     2.952425E-17
  6    7.5000   -1.049105E-05    9.150023E-20       1.009749E-07    -2.978771E-06     2.298466E-17
  7    9.0000   -1.049105E-05    0.                 1.009749E-07    -2.978771E-06     0.

ROW   9.  X=     .8000
COL      Y             W                  V                 U              BETAX            BETAY
  1    0.0000   -7.552051E-06    0.                 2.374786E-07     2.985069E-05     0.
  2    1.5000   -7.552051E-06   -9.734609E-21       2.374786E-07     2.985069E-05    -1.549469E-18
  3    3.0000   -7.552051E-06   -2.257603E-20       2.374786E-07     2.985069E-05    -7.762290E-18
  4    4.5000   -7.552051E-06   -3.163051E-20       2.374786E-07     2.985069E-05    -1.605067E-17
  5    6.0000   -7.552051E-06   -3.110207E-20       2.374786E-07     2.985069E-05    -1.760602E-17
  6    7.5000   -7.552051E-06   -1.750681E-20       2.374786E-07     2.985069E-05     1.002437E-20
  7    9.0000   -7.552051E-06    0.                 2.374786E-07     2.985069E-05     0.
```

Fig. 7-24 (Cont.)

```
ROW  11.  X=     1.0000
COL      Y           W               V              U              BETAX          BETAY
  1    0.0000   0.              0.             -1.058791E-22   4.517782E-24   0.
  2    1.5000   0.              9.629650E-35   -1.058791E-22   4.517782E-24  -5.341569E-35
  3    3.0000   0.              4.814825E-35   -2.117582E-22   9.035564E-24  -2.670784E-35
  4    4.5000   0.              4.814825E-35   -1.058791E-22   4.517782E-24  -2.670784E-35
  5    6.0000   0.              4.814825E-35   -1.058791E-22   4.517782E-24  -2.670784E-35
  6    7.5000   0.              0.             -1.058791E-22   4.517782E-24   0.
  7    9.0000   0.              0.             -1.058791E-22   4.517782E-24   0.

ROW   1.  X=     0.0000
COL      Y           NX              NY             NXY             MX             MY             MXY
  1    0.0000  -3.802773E-01  -1.140832E-01   0.              8.456916E-03   2.549090E-03   0.
  2    1.5000  -3.802773E-01  -1.140832E-01   2.185640E-14    8.456916E-03   2.549090E-03   1.517811E-18
  3    3.0000  -3.802773E-01  -1.140832E-01   7.228992E-14    8.456916E-03   2.549090E-03   5.020133E-18
  4    4.5000  -3.802773E-01  -1.140832E-01   4.036359E-14    8.456916E-03   2.549090E-03   2.803027E-18
  5    6.0000  -3.802773E-01  -1.140832E-01   5.955934E-14    8.456916E-03   2.549090E-03   4.136066E-18
  6    7.5000  -3.802773E-01  -1.140832E-01   1.041022E-14    8.456916E-03   2.549090E-03   7.229323E-19
  7    9.0000  -3.802773E-01  -1.140832E-01   0.              8.456916E-03   2.549090E-03   0.

ROW   6.  X=      .5000
COL      Y           NX              NY             NXY             MX             MY             MXY
  1    0.0000  -4.639973E-01  -1.715589E+00   0.             -5.670286E-04  -1.643041E-04   0.
  2    1.5000  -4.639973E-01  -1.715589E+00  -5.438822E-16   -5.670286E-04  -1.643041E-04  -1.310505E-16
  3    3.0000  -4.639973E-01  -1.715589E+00  -2.185497E-14   -5.670286E-04  -1.643041E-04   4.015542E-16
  4    4.5000  -4.639973E-01  -1.715589E+00  -1.363769E-14   -5.670286E-04  -1.643041E-04   1.135062E-15
  5    6.0000  -4.639973E-01  -1.715589E+00  -3.551571E-14   -5.670286E-04  -1.643041E-04   1.524379E-15
  6    7.5000  -4.639973E-01  -1.715589E+00  -1.769912E-14   -5.670286E-04  -1.643041E-04   1.236675E-15
  7    9.0000  -4.639973E-01  -1.715589E+00   0.             -5.670286E-04  -1.643041E-04   0.

ROW   9.  X=      .8000
COL      Y           NX              NY             NXY             MX             MY             MXY
  1    0.0000  -5.260554E-01  -1.228732E+00   0.             -2.235094E-03  -7.323348E-04   0.
  2    1.5000  -5.260554E-01  -1.228732E+00  -3.451942E-14   -2.235094E-03  -7.323348E-04   1.228989E-15
  3    3.0000  -5.260554E-01  -1.228732E+00  -2.998507E-14   -2.235094E-03  -7.323348E-04   1.893459E-15
  4    4.5000  -5.260554E-01  -1.228732E+00  -6.310357E-14   -2.235094E-03  -7.323348E-04   1.578974E-15
  5    6.0000  -5.260554E-01  -1.228732E+00  -3.266651E-14   -2.235094E-03  -7.323348E-04   9.024506E-16
  6    7.5000  -5.260554E-01  -1.228732E+00  -2.882918E-14   -2.235094E-03  -7.323348E-04   7.342444E-16
  7    9.0000  -5.260554E-01  -1.228732E+00   0.             -2.235094E-03  -7.323348E-04   0.

ROW  11.  X=     1.0000
COL      Y           NX              NY             NXY             MX             MY             MXY
  1    0.0000  -5.285066E-01  -1.585520E-01   0.              9.496431E-03   2.862824E-03   0.
  2    1.5000  -5.285066E-01  -1.585520E-01   3.609324E-14    9.496431E-03   2.862824E-03   2.085513E-18
  3    3.0000  -5.285066E-01  -1.585520E-01   7.805472E-14    9.496431E-03   2.862824E-03   4.510101E-18
  4    4.5000  -5.285066E-01  -1.585520E-01   5.483356E-14    9.496431E-03   2.862824E-03   3.168353E-18
  5    6.0000  -5.285066E-01  -1.585520E-01   6.498941E-14    9.496431E-03   2.862824E-03   3.755171E-18
  6    7.5000  -5.285066E-01  -1.585520E-01   2.811968E-14    9.496431E-03   2.862824E-03   1.624791E-18
  7    9.0000  -5.285066E-01  -1.585520E-01   0.              9.496431E-03   2.862824E-03   0.

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CFD2
    CCC(I,1)            CCC(I,2)            CCC(I,3)            CCC(I,4)            CCC(I,5)            CCC(I,6)

     2.747253E+05        0.                  0.                  0.                  0.                  0.
     8.241758E+04        2.747253E+05        0.                  0.                  0.                  0.
     0.                  0.                  9.615385E+04        0.                  0.                  0.
     0.                  0.                  0.                  1.430861E+01        0.                  0.
     0.                  0.                  0.                  4.292582E+00        1.430861E+01        0.
     0.                  0.                  0.                  0.                  0.                  5.008813E+00

ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=   12.877.  NR OF IO REQUESTS (TAPE2)=    97.     WORDS USED (TAPE2)=   77113.    WORDS TRANSFERRED (TAPE2)=      431039
```

Fig. 7-24 (Cont.)

```
DETERMINANT OF STIFFNESS MATRIX=   7.9886266E+02*10.**   450.   NUMBER OF NEGATIVE ROOTS =   0
      77 NODES.       351 EQUATIONS.   MAXIMUM BAND WIDTH =        60
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=   13.678.  NR OF IO REQUESTS (TAPE2)=   183.      WORDS USED (TAPE2)=   77113.    WORDS TRANSFERRED (TAPE2)=      460811

EIGENVALUE SHIFT=  -0.            .     NUMBER OF NEGATIVE   ROOTS=       0
```

ITERATION	EIGENVALUE (RAYLEIGH QUOTIENT)	EIGENVALUE (ACCELERATED ESTIMATE)
0	6.5353416E+01	4.5474735E-13
1	7.1997007E+02	-7.2481512E+00
2	7.0432725E+02	7.0440631E+02
3	7.0281230E+02	7.0271208E+02
4	7.0267223E+02	7.0265397E+02
5	7.0265532E+02	7.0265300E+02

THE BUCKLING LOAD BASED ON LINEAR BIFURCATION THEORY IS 7.026530E+02 TIMES THE STARTING LOAD.

ROW 1. X= 0.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	0.	0.	0.	0.
2	1.5000	0.	0.	0.	0.	0.
3	3.0000	0.	6.938894E-18	-1.734723E-18	1.284980E-19	-4.625929E-18
4	4.5000	0.	6.938894E-18	-3.469447E-18	2.569961E-19	-4.625929E-18
5	6.0000	0.	1.040834E-17	-6.938894E-18	5.139921E-19	-6.938894E-18
6	7.5000	0.	6.938894E-18	0.	0.	-4.625929E-18
7	9.0000	0.	0.	0.	0.	0.

ROW 6. X= .5000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	9.095535E-02	0.	0.	6.198254E+00
2	1.5000	2.533416E-01	8.785593E-02	3.078513E-04	2.105412E-02	5.986950E+00
3	3.0000	5.000351E-01	7.876903E-02	5.947289E-04	4.066521E-02	5.367499E+00
4	4.5000	7.071398E-01	6.431410E-02	8.410651E-04	5.749337E-02	4.382263E+00
5	6.0000	8.660456E-01	4.547656E-02	1.030129E-03	7.039517E-02	3.098529E+00
6	7.5000	9.659319E-01	2.354026E-02	1.148971E-03	7.849962E-02	1.603841E+00
7	9.0000	1.000000E+00	0.	1.189506E-03	8.126272E-02	0.

ROW 9. X= .8000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	4.308898E-02	0.	0.	2.656389E+00
2	1.5000	1.174534E-01	4.162069E-02	1.804772E-03	-7.083977E-01	2.565840E+00
3	3.0000	2.268936E-01	3.731591E-02	3.486524E-03	-1.368398E+00	2.300382E+00
4	4.5000	3.208700E-01	3.046812E-02	4.930634E-03	-1.935159E+00	1.878157E+00
5	6.0000	3.929766E-01	2.154408E-02	6.038701E-03	-2.370017E+00	1.327992E+00
6	7.5000	4.383024E-01	1.115198E-02	6.735241E-03	-2.643362E+00	6.873920E-01
7	9.0000	4.537618E-01	0.	6.972814E-03	-2.736592E+00	0.

ROW 11. X= 1.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	0.	-1.387779E-17	0.	0.	7.698012E-18
2	1.5000	0.	-2.775558E-17	-1.734723E-18	7.401934E-20	1.539502E-17
3	3.0000	0.	0.	-1.734723E-18	7.401934E-20	0.
4	4.5000	0.	0.	-3.469447E-18	1.480387E-19	0.
5	6.0000	0.	-3.469447E-18	-3.469447E-18	1.480387E-19	1.924503E-18
6	7.5000	0.	-8.673617E-18	-3.469447E-18	1.480387E-19	4.811257E-18
7	9.0000	0.	0.	-3.469447E-18	1.480387E-19	0.

```
CP SECONDS=   15.594.  NR OF IO REQUESTS (TAPE2)=   155.      WORDS USED (TAPE2)=   84612.    WORDS TRANSFERRED (TAPE2)=      723938
```

Fig. 7-24 (Cont.)

7.9 SAMPLE CASE 9 – FIBERWOUND CYLINDER

The critical axial line load according to bifurcation theory is to be determined for a circular cylindrical shell built up from 3 fiber-wound layers and clamped at both ends. The geometry of the shell is shown in Fig. 7-25.

Analysis with BOSOR4 (Ref. 15) shows that for this shell, the critical axial line load is 500 lb/in. for a mode with 11 circumferential waves. Therefore, a shell covering one half the length and 1/44 of the circumference can be analyzed.

The meridional boundary conditions are the same as for Sample Case 4. The input cards associated with this case are displayed in Fig. 7-26. Portions of the output are presented in Fig. 7-27.

The critical axial line load determined here is 609 lb/in. Note that the discrepancy in the solution is due to the coarseness of the grid in the longitudinal direction, selected here to reduce the run time.

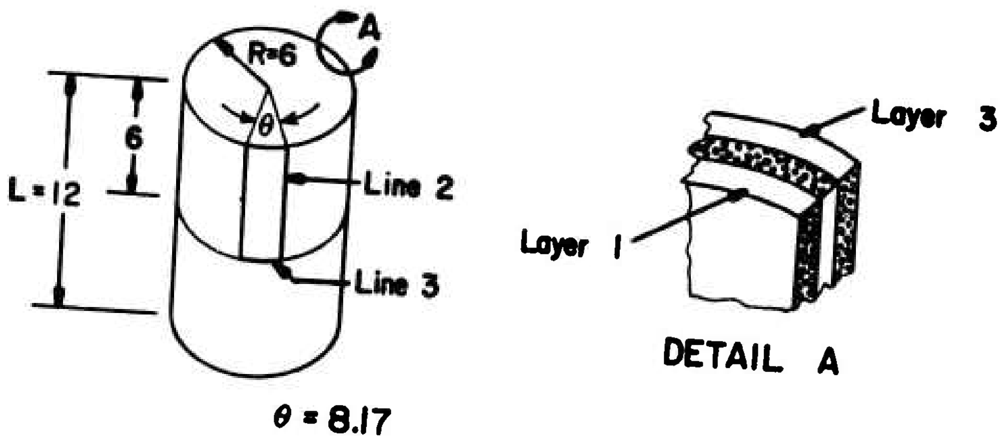


Fig. 7-25 Sample Case 9 – Fiberwound Cylinder

SAMPLE CASE 9
INPUT

```
    SAMPLE CASE 9 - CYLINDER FIBERWOUND                                      C-1
       1     1     1
 6.0          8.17        6.0                                               G-1
      19     9                                                               G-2
       0     4     4     4                                                   D-1
       0     0     1     0                                                   B-1
       1     0  1.        1.           1.                                    B-2
 0.           0.          1.                0      0      2      1      0    L-1
             500.0              1      2    20                               L-2
       2     4     4     5                                                   P-1A
       0     0     1     4     0       1     1                               P-1A1
 0.           2.          4.          6.                                     O-1
      4                                                                      O-2
 12400000.  500000.0   0.22         0.35        0.0                          M-1
 0.0115       .32      90.0         0.3                                 3    M-2D1
 0.013        .32      0.0          0.3                                    } M-2D2
 0.0115       .32      90.0         0.3
```

Fig. 7-26 Display of Input Cards for Sample Case 9

SAMPLE CASE 9 – OUTPUT

```
   SAMPLE CASE 9 - CYLINDER FIBERWOUND
BUCKLING ANALYSIS.              1 LOAD PATTERNS.

TYPE OF SURFACE IS CYLINDER

SURFACE CONSTANTS =  6.0000000E+00, 0.1700000E+00, 6.0000000E+00,

 BLANK COMMON ARRAY WORKING SPACE=      15000

  FINITE DIFFERENCE MESH,  19 ROWS,    9 COLUMNS.  MESH SPACING. H=    .3333, K=   1.0212

NR41=  -0, NR42= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS SET BY IFREE . IFREE = 0, 0, 1, 0,
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =      1

 USER-LOAD FLAG =      0,  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP =  1.0000000E+00,  MAXIMUM LOAD =  1.0000000E+00

       PZ             PY              PX          JZ    JY    JX  ROW  COL
 3.             0.            1.0000000E+00      0     0     2    1    0

BOUNDARY CONDITIONS FOR BUCKLING DISPLACEMENTS
BOUNDARY CONDITION AT LINE 1 IS   CLAMPED
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  ANTI-METRIC

 ISHIFT    ITERAT    SHIFT
  2         20        5.000000E+02

IPR=     4 IPY=     0 IPRD=     1 IPRS=     1 IPLOT=    -0

IWALL=     4,  ISTRI=   -0,  NRING=   -0,  IP=   -0,  IM=   -0,  JM=   -0

ANALYSIS IS FOR A FIBER REINFORCED SHELL.

FIBER  MODULUS= 1.2430000E+07      FIBER  POISSON RATIO= 2.2900000E-01
MATRIX MODULUS= 5.0000000E+05      MATRIX POISSON RATIO= 3.5000000E-01

               LAYER          THICKNESS           MATRIX CONTENT        WINDING ANGLE    CONTIGUITY FACTOR
                 1         1.1500000E-02        3.2000000E-01       9.0000000E+01      3.0000000E-01
                 2         1.3000000E-02        3.2000000E-01       0.                 3.0000000E-01
                 3         1.1500000E-02        3.2000000E-01       9.0000000E+01      3.0000000E-01

THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CFRN
     CCC(I,1)             CCC(I,2)             CCC(I,3)             CCC(I,4)             CCC(I,5)             CCC(I,6)

   1.759966E+05        0.                   0.                   0.                   0.                   0.
   2.651290E+06        2.371700E+05         0.                   0.                   0.                   0.
   3.349317E-05        2.179192E-04         5.042475E+04         0.                   0.                   0.
   2.332494E-11        3.132231E-12         3.380132E-21         1.176895E+01         0.                   0.
   3.183231E-12        2.919343E-11         2.032879E-20         2.647382E+00         3.296104E+01         0.
   3.380132E-21        2.032879E-20         6.366463E-12         4.911942E-09         3.510259E-08         5.445873E+00
```

Fig. 7-27 Excerpt of Output for Sample Case 9

```
 CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=    2.606.  NR OF IO REQUESTS (TAPE2)=    13.      WORDS USED (TAPE2)=  39858.     WORDS TRANSFERRED (TAPE2)=       43954

 FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=    6.751.  NR OF IO REQUESTS (TAPE2)=    29.      WORDS USED (TAPE2)=  71537.     WORDS TRANSFERRED (TAPE2)=      111408

 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=    7.612.  NR OF IO REQUESTS (TAPE2)=    43.      WORDS USED (TAPE2)=  98109.     WORDS TRANSFERRED (TAPE2)=      194183


DETERMINANT OF STIFFNESS MATRIX=    3.4417076E+05*10.**  1120.    NUMBER OF NEGATIVE ROOTS =   0
      171 NODES,        693 EQUATIONS.   MAXIMUM BAND WIDTH =         72
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=   10.272.  NR OF IO REQUESTS (TAPE2)=    53.      WORDS USED (TAPE2)=  98109.     WORDS TRANSFERRED (TAPE2)=      249375


LINEAR SOLUTION.  PA=   1.000000E+00,    PB=   0.
ROW 18,   X=      5.667, AXIAL LOAD=         -8.555604E-01,BENDING MOMENT=      0.

 ROW    1,  X=     0.0000
  COL        Y               W                 V                 U             BETAX              BETAY
    1      .0356      0.                0.                3.437020E-05    0.                 0.
    2      .7659      0.                9.860761E-32      3.437020E-05    0.                 -1.643460E-32
    3     1.5319      0.                1.972152E-31      3.437020E-05    0.                 -3.286920E-32
    4     2.5531      0.                1.972152E-31      3.437020E-05    0.                 -3.286920E-32
    5     3.5744      0.                1.972152E-31      3.437020E-05    0.                 -3.286920E-32
    6     4.5956      0.                1.972152E-31      3.437020E-05    0.                 -3.286920E-32
    7     5.6169      0.                9.860761E-32      3.437020E-05    0.                 -1.643460E-32
    8     6.6381      0.                9.860761E-32      3.437020E-05    0.                 -1.643460E-32
    9     7.6594      0.                0.                3.437020E-05    0.                 0.

 ROW    7,  X=     2.0000
  COL        Y               W                 V                 U             BETAX              BETAY
    1      .0356      3.552671E-06      0.                2.292748E-05    -3.998728E-09      0.
    2      .7659      3.552671E-06      -1.130220E-18     2.292748E-05    -3.998728E-09      -3.804783E-17
    3     1.5319      3.552671E-06      -1.881868E-18     2.292748E-05    -3.998728E-09      -6.596725E-17
    4     2.5531      3.552671E-06      -2.289151E-18     2.292748E-05    -3.998728E-09      -8.042768E-17
    5     3.5744      3.552671E-06      -2.422170E-18     2.292748E-05    -3.998728E-09      -8.560968E-17
    6     4.5956      3.552671E-06      -2.253538E-18     2.292748E-05    -3.998728E-09      -7.949395E-17
    7     5.6169      3.552671E-06      -1.827553E-18     2.292748E-05    -3.998728E-09      -6.315738E-17
    8     6.6381      3.552671E-06      -1.089973E-18     2.292748E-05    -3.998728E-09      -3.581400E-17
    9     7.6594      3.552671E-06      0.                2.292748E-05    -3.998728E-09      0.

 ROW   13,  X=     4.0000
  COL        Y               W                 V                 U             BETAX              BETAY
    1      .0356      3.554519E-06      0.                1.146375E-05    -4.967574E-12      0.
    2      .7659      3.554519E-06      -7.514093E-19     1.146375E-05    -4.967578E-12      -2.719666E-17
    3     1.5319      3.554519E-06      -1.270713E-18     1.146375E-05    -4.967589E-12      -4.713171E-17
    4     2.5531      3.554519E-06      -1.571942E-18     1.146375E-05    -4.967605E-12      -5.691160E-17
    5     3.5744      3.554519E-06      -1.672886E-18     1.146375E-05    -4.967625E-12      -6.072563E-17
    6     4.5956      3.554519E-06      -1.555324E-18     1.146375E-05    -4.967645E-12      -5.546877E-17
    7     5.6169      3.554519E-06      -1.229569E-18     1.146375E-05    -4.967662E-12      -4.301860E-17
    8     6.6381      3.554519E-06      -7.072619E-19     1.146375E-05    -4.967673E-12      -2.402369E-17
    9     7.6594      3.554519E-06      0.                1.146375E-05    -4.967678E-12      0.
```

Fig. 7-27 (Cont.)

ROW 19. X= 6.0000

COL	Y	W	V	U	BETAX	BETAY
1	.0356	3.554520E-06	0.	-1.694056E-21	0.	0.
2	.7659	3.554520E-06	1.729861E-17	-3.388132E-21	0.	6.748878E-16
3	1.5319	3.554520E-06	3.182964E-17	0.	0.	1.238709E-15
4	2.5531	3.554520E-06	4.126751E-17	-1.694066E-21	0.	1.609740E-15
5	3.5744	3.554520E-06	4.450049E-17	-1.694066E-21	0.	1.738004E-15
6	4.5956	3.554520E-06	4.127759E-17	-3.388132E-21	0.	1.610461E-15
7	5.6169	3.554520E-06	3.184066E-17	0.	0.	1.240658E-15
8	6.6381	3.554520E-06	1.730847E-17	-1.694066E-21	0.	6.760427E-16
9	7.6594	3.554520E-06	0.	-1.694066E-21	0.	0.

ROW 1. X= 0.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	.0356	-1.000000E+00	-1.384931E-01	-1.722812E-10	-5.321557E-04	-1.197065E-04	-2.221029E-13
2	.7659	-1.000000E+00	-1.384931E-01	-3.090443E-11	-5.321557E-04	-1.197065E-04	-2.217535E-13
3	1.5319	-1.000000E+00	-1.384931E-01	-4.538194E-11	-5.321557E-04	-1.197065E-04	-2.215019E-13
4	2.5531	-1.000000E+00	-1.384931E-01	-4.907205E-11	-5.321557E-04	-1.197065E-04	-2.213632E-13
5	3.5744	-1.000000E+00	-1.384931E-01	-4.949414E-11	-5.321557E-04	-1.197065E-04	-2.213198E-13
6	4.5956	-1.000000E+00	-1.384931E-01	-4.880202E-11	-5.321557E-04	-1.197065E-04	-2.213635E-13
7	5.6169	-1.000000E+00	-1.384931E-01	-4.533403E-11	-5.321557E-04	-1.197065E-04	-2.215015E-13
8	6.6381	-1.000000E+00	-1.384931E-01	-3.087433E-11	-5.321557E-04	-1.197065E-04	-2.217536E-13
9	7.6594	-1.000000E+00	-1.384931E-01	-1.722812E-10	-5.321557E-04	-1.197065E-04	-2.221029E-13

ROW 7. X= 2.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	.0356	-1.000000E+00	-7.201588E-05	-4.574975E-11	-5.140938E-07	-1.156435[illegible]	[illegible]45645E-16
2	.7659	-1.000000E+00	-7.201598E-05	8.933659E-13	-5.140938E-07	-1.156435[illegible]	[illegible]971E-15
3	1.5319	-1.000000E+00	-7.201598E-05	1.257368E-12	-5.140938E-07	-1.156435E-[illegible]	[illegible]6020E-15
4	2.5531	-1.000000E+00	-7.201598E-05	1.371747E-12	-5.140938E-07	-1.156435E-07	[illegible]E-15
5	3.5744	-1.000000E+00	-7.201598E-05	1.405595E-12	-5.140938E-07	-1.156435E-07	[illegible]E-15
6	4.5956	-1.000000E+00	-7.201588E-05	1.235826E-12	-5.140938E-07	-1.156435E-07	-2.702595E-15
7	5.6169	-1.000000E+00	-7.201588E-05	1.108771E-12	-5.140938E-07	-1.156435E-07	-2.093971E-15
8	6.6381	-1.000000E+00	-7.201589E-05	7.750163E-13	-5.140938E-07	-1.156435E-07	-1.227698E-15
9	7.6594	-1.000000E+00	-7.201588E-05	-4.574975E-11	-5.140938E-07	-1.156435E-07	-2.145645E-16

ROW 13. X= 4.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	.0356	-1.000000E+00	-3.120110E-08	-4.568394E-11	-4.147727E-10	-9.329248E-11	-1.731019E-19
2	.7659	-1.000000E+00	-3.120067E-08	-3.276793E-13	-4.147728E-10	-9.329409E-11	7.453865E-16
3	1.5319	-1.000000E+00	-3.120042E-08	-6.516528E-13	-4.147728E-10	-9.329699E-11	1.388517E-15
4	2.5531	-1.000000E+00	-3.120027E-08	-7.775968E-13	-4.147727E-10	-9.329960E-11	1.837611E-15
5	3.5744	-1.000000E+00	-3.120017E-08	-8.485661E-13	-4.147726E-10	-9.330185E-11	2.012154E-15
6	4.5956	-1.000000E+00	-3.119998E-08	-9.027535E-13	-4.147724E-10	-9.330406E-11	1.893118E-15
7	5.6169	-1.000000E+00	-3.119988E-08	-6.444433E-13	-4.147724E-10	-9.330512E-11	1.454514E-15
8	6.6381	-1.000000E+00	-3.119962E-08	-4.529438E-13	-4.147724E-10	-9.330822E-11	7.908920E-16
9	7.6594	-1.000000E+00	-3.119932E-08	-4.568394E-11	-4.147725E-10	-9.330947E-11	-1.731196E-19

ROW 19. X= 6.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	.0356	-1.000000E+00	1.509326E-11	-4.568392E-11	-6.781387E-13	-3.656658E-13	-5.068878E-22
2	.7659	-1.000000E+00	1.079903E-11	-4.568392E-11	-6.718710E-13	-3.462600E-13	-4.853321E-22
3	1.5319	-1.000000E+00	1.165178E-12	-4.568392E-11	-6.538663E-13	-2.930657E-13	-4.261945E-22
4	2.5531	-1.000000E+00	-8.556711E-12	-4.568392E-11	-6.276354E-13	-2.189382E-13	-3.436145E-22
5	3.5744	-1.000000E+00	-1.792166E-11	-4.568392E-11	-5.975650E-13	-1.347948E-13	-2.497820E-22
6	4.5956	-1.000000E+00	-2.725375E-11	-4.568392E-11	-5.674450E-13	-4.969156E-14	-1.549322E-22
7	5.6169	-1.000000E+00	-3.702438E-11	-4.568392E-11	-5.414836E-13	2.456859E-14	-7.222540E-23
8	6.6381	-1.000000E+00	-4.660450E-11	-4.568392E-11	-5.235564E-13	7.779658E-14	-1.306544E-23
9	7.6594	-1.000000E+00	-5.095143E-11	-4.568392E-11	-5.173135E-13	9.714077E-14	8.416895E-24

Fig. 7-27 (Cont.)

```
THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CF04
     CCC(I,1)             CCC(I,2)             CCC(I,3)             CCC(I,4)             CCC(I,5)             CCC(I,6)

   1.7[illegible]966E+05    0.                   0.                   0.                   0.                   0.
   2.[illegible]E+04        2.371700E+05         0.                   0.                   0.                   0.
   3.0[illegible]171E-05    2.179102E-04         5.042475E+04         0.                   0.                   0.
   2.00[illegible]E-11      3.183231E-12         3/386132E-21         1.176895E+01         0.                   0.
   3.1[illegible]E-12       2.910343E-11         2.037079E-20         2.647382E+00         3.296104E+01         0.
   3.[illegible]E-21        2.037079E-20         6.366463E-12         4.911942E-09         3.510299E-08         5.445073E+00

 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=   1[illegible].230.   NR OF IO REQUESTS (TAPE2)=   122.      WORDS USED (TAPE2)=  100157.     WORDS TRANSFERRED (TAPE2)=     560509


DETERMINANT OF STIFFNESS MATRIX=   6.4129400E+06*10.**  1050.   NUMBER OF NEGATIVE ROOTS =   0
      171 NODES.        643 EQUATIONS.   MAXIMUM BAND WIDTH =         72
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=   21.647.   NR OF IO REQUESTS (TAPE2)=   132.      WORDS USED (TAPE2)=  100157.     WORDS TRANSFERRED (TAPE2)=     615777

EIGENVALUE SHIFT=    5.0000000E+02,      NUMBER OF NEGATIVE   ROOTS=        0

ITERATION          EIGENVALUE                   EIGENVALUE
               (RAYLEIGH QUOTIENT)          (ACCELERATED ESTIMATE)
     0           5.6595465E+02                 5.0000000E+02
     1           7.2172044E+02                 4.5156488E+02
     2           6.1737774E+02                 6.5923499E+02
     3           6.1110973E+02                 6.1059590E+02
     4           6.1057713E+02                 6.1046401E+02
     5           6.1023144E+02                 6.0993023E+02
     6           6.0990477E+02                 6.0622630E+02
     7           6.0476627E+02                 6.0821109E+02
     8           6.0359225E+02                 6.0865598E+02
     9           6.0943261E+02                 6.0876248E+02
    10           6.0931031E+02                 6.0879141E+02
    11           6.0921194E+02                 6.0879701E+02
    12           6.0913233E+02                 6.0879487E+02
    13           6.0906774E+02                 6.0879031E+02
    14           6.0901520E+02                 6.0878531E+02
    15           6.0897227E+02                 6.0878063E+02
    16           6.0893705E+02                 6.0877654E+02
    17           6.0890807E+02                 6.0877309E+02
    18           6.0888417E+02                 6.0877029E+02
    19           6.0886426E+02                 6.0876795E+02
    20           6.0884775E+02                 6.0876612E+02
 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=   33.880.   NR OF IO REQUESTS (TAPE2)=   410.      WORDS USED (TAPE2)=  121110.     WORDS TRANSFERRED (TAPE2)=    2303624
```

Fig. 7-27 (Cont.)

DETERMINANT OF STIFFNESS MATRIX= 2.6674315E+05*10.** 1060. NUMBER OF NEGATIVE ROOTS = 0
171 NODES. 693 EQUATIONS. MAXIMUM BAND WIDTH = 72
MATRIX DECOMPOSITION COMPLETED.
CP SECONDS= 36.256. NR OF IO REQUESTS (TAPE2)= 420. WORDS USED (TAPE2)= 121110. WORDS TRANSFERRED (TAPE2)= 2358816

EIGENVALUE SHIFT= 5.9232358E+02. NUMBER OF NEGATIVE ROOTS= 0

ITERATION	EIGENVALUE (RAYLEIGH QUOTIENT)	EIGENVALUE (ACCELERATED ESTIMATE)
0	6.0[illegible]E+02	6.0887306E+02
1	6.0[illegible]E+02	6.0875930E+02
2	6.0[illegible]E+02	6.0876049E+02
3	6.0876374E+02	6.0876010E+02

THE BUCKLING LOAD BASED ON LINEAR BIFURCATION THEORY IS 6.087601E+02 TIMES THE STARTING LOAD.

ROW 1. X= 0.0000

COL	Y	W	V	U	BETAX	BETAY
1	.0356	0.	3.469447E-18	0.	0.	-5.782412E-19
2	.7659	0.	6.938894E-18	5.421011E-20	0.	-1.156482E-18
3	1.5319	0.	1.734723E-18	1.084202E-19	0.	-2.891206E-19
4	2.5531	0.	3.469447E-18	1.084202E-19	0.	-5.782412E-19
5	3.5744	0.	0.	2.168404E-19	0.	0.
6	4.5956	0.	0.	0.	0.	0.
7	5.6169	0.	8.673617E-19	0.	0.	-1.445603E-19
8	6.6381	0.	0.	2.168404E-19	0.	0.
9	7.6594	0.	0.	2.168404E-19	0.	0.

ROW 7. X= 2.0000

COL	Y	W	V	U	BETAX	BETAY
1	.0356	0.	-2.972354E-02	0.	0.	-6.235909E-01
2	.7659	-6.721976E-02	-2.915241E-02	4.482716E-04	-1.189155E-01	-6.116088E-01
3	1.5319	-1.318563E-01	-2.746097E-02	8.793164E-04	-2.332612E-01	-5.761229E-01
4	2.5531	-1.914257E-01	-2.471422E-02	1.276569E-03	-3.386428E-01	-5.184969E-01
5	3.5744	-2.4[illegible]E-01	-2.101772E-02	1.624765E-03	-4.310105E-01	-4.409454E-01
6	4.5956	-2.4[illegible]E-01	-1.651352E-02	1.910521E-03	-5.068147E-01	-3.464486E-01
7	5.6169	-3.1[illegible]E-01	-1.137471E-02	2.122857E-03	-5.631423E-01	-2.385379E-01
8	6.6381	-3.3[illegible]E-01	-5.798775E-03	2.253613E-03	-5.978287E-01	-1.216566E-01
9	7.6594	-3.445571E-01	0.	2.297754E-03	-6.095409E-01	0.

ROW 13. X= 4.0000

COL	Y	W	V	U	BETAX	BETAY
1	.0356	0.	6.456935E-02	0.	0.	1.453835E+00
2	.7659	1.5[illegible]E-01	6.332967E-02	-6.006177E-04	1.097176E-01	1.425900E+00
3	1.5319	3.072435E-01	5.965430E-02	-1.178154E-03	2.152188E-01	1.343169E+00
4	2.5531	4.460485E-01	5.368745E-02	-1.710414E-03	3.124492E-01	1.208920E+00
5	3.5744	5.677120E-01	4.565742E-02	-2.176945E-03	3.976724E-01	1.028017E+00
6	4.5956	6.675597E-01	3.587251E-02	-2.559816E-03	4.676133E-01	8.077077E-01
7	5.6169	7.417515E-01	2.470362E-02	-2.844315E-03	5.195441E-01	5.563588E-01
8	6.6381	7.874312E-01	1.259586E-02	-3.019509E-03	5.515875E-01	2.836292E-01
9	7.6594	8.028561E-01	0.	-3.078665E-03	5.623938E-01	0.

ROW 19. X= 6.0000

COL	Y	W	V	U	BETAX	BETAY
1	.0356	0.	-7.947435E-02	0.	0.	-1.810965E+00
2	.7659	-1.900303E-01	-7.794727E-02	-1.734723E-18	0.	-1.776168E+00
3	1.5319	-3.828434E-01	-7.342472E-02	0.	0.	-1.673114E+00
4	2.5531	-5.557025E-01	-6.608051E-02	-3.469447E-18	0.	-1.505763E+00
5	3.5744	-7.071059E-01	-5.615695E-02	-3.469447E-18	0.	-1.280546E+00
6	4.5956	-8.3[illegible]E-01	-4.415358E-02	-6.938894E-18	0.	-1.006118E+00
7	5.6169	-9.[illegible]E-01	-3.041352E-02	-1.387779E-17	0.	-6.930264E-01
8	6.6381	-9.[illegible]E-01	-1.550458E-02	0.	0.	-3.533018E-01
9	7.6594	-1.000000E+00	0.	0.	0.	0.

CP SECONDS= 37.434. NR OF IO REQUESTS (TAPE2)= 466. WORDS USED (TAPE2)= 121110. WORDS TRANSFERRED (TAPE2)= 2635787

Fig. 7-27 (Cont.)

7.10 SAMPLE CASE 10 – THERMAL LOADS

The critical load combination according to bifurcation theory will be determined for a simply supported circular cylindrical shell loaded with axial compressive line load and subjected to temperature distribution that varies linearly along the length and is constant circumferentially.

Analysis with BOSOR4 (Re 15) shows that for this shell, the lowest eigenvalue of 3.156 occurs in conjunction with 26 circumferential waves. Therefore, a shell covering one half the length and 1/52 of the circumference can be analyzed.

User-written subroutine MATER is used here to introduce the material properties and the temperature distribution. However, this particular example could be solved using user-written subroutine TEMP and input card M-2B. The geometry and temperature variation of the shell are shown in Fig. 7-28. Subroutine MATER and the input cards associated with this case are shown in Fig. 7-29 and Fig. 7-30, respectively. Portions of the output are presented in Fig. 7-31. The critical load for this case is the eigenvalue times (STLD*PP), where PP is the base load composed of the axial line load (1 lb/in.) plus the thermal loads due to the temperature distribution as shown in Fig. 7-28. The lowest eigenvalue determined here is 3.269.

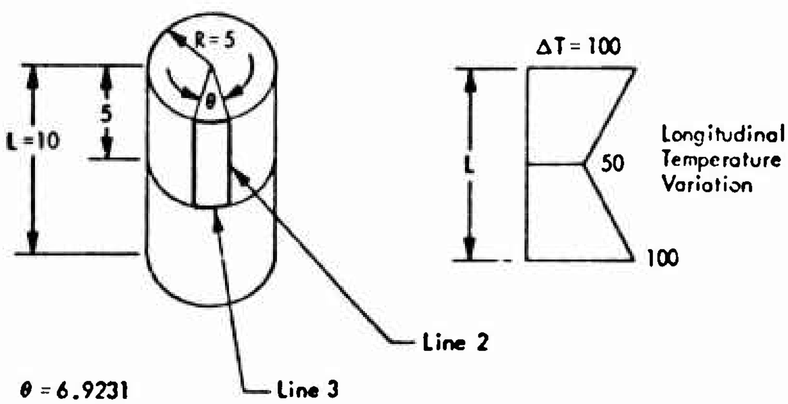


Fig. 7-28 Sample Case 10 – Thermal Loading

```
      SUBROUTINE MATER (X,Y,IP,TDEG,EX,EY,U,G,A1,A2)
      DIMENSION TDEG(IP),EX(IP),EY(IP),U(IP),G(IP),A1(IP),A2(IP)
      COMMON /OFST/ TD,Z
C     TD AND Z MUST BE SET BY THE USER
C     TD = TOTAL THICKNESS OF SHELL (TD = AT)
C     Z = DISTANCE FROM REFERENCE SURFACE TO MIDSURFACE OF SHELL WALL
      AT=0.03
      Z=0.
      TD=AT
      DO 1 L=1,IP
      C=1.
      TDEG(L)=50.*(2.-X/10.)
      EX(L)=C*10000000.
      EY(L)=C*10000000.
      U(L)=0.3
      G(L)=C*4000000.
      A1(L)=0.0001
      A2(L)=0.0001
    1 CONTINUE
      RETURN
      END
```

Fig. 7-29 Subroutine MATER for Sample Case 10

SAMPLE CASE 10
INPUT

```
  SAMPLE CASE 10 - THERMAL LOADS (IWALL=8)                        C-1
   1    1    1                                                    G-1
5.         6.9231     5.                                          G-2
   0    6                                                         D-1
   3                                                              D-2
1.         3.         1.                                          D-3
  10   10   10                                                    D-4
   1    4    4    4                                               B-1
   1    0 1.                                                      L-1
0.         0.         1.         0    0    2    1    0            L-2
                              1   16                              P-1A
   0    0    1    4    0    1    1                                O-1
.3         1.9        3.4        5.                               O-2
   8    0    0    3                                               M-1
```

Fig. 7-30 Display of Input Cards for Sample Case 10

SAMPLE CASE 10 – OUTPUT

```
      SAMPLE CASE 10 - THERMAL LOADS (IWALL=0)
BUCKLING ANALYSIS.             1 LOAD PATTERNS.

TYPE OF SURFACE IS CYLINDER

SURFACE CONSTANTS =  5.00000000E+00, 6.92310000E+00, 5.00000000E+00,

STATION NO.   COORDINATE
     1           0.0000
     2            .1000
     3            .2000
     4            .3000
     5            .4000
     6            .5000
     7            .6000
     8            .7000
     9            .8000
    10            .9000
    11           1.0000
    12           1.3000
    13           1.6000
    14           1.9000
    15           2.2000
    16           2.5000
    17           2.8000
    18           3.1000
    19           3.4000
    20           3.7000
    21           4.0000
    22           4.1000
    23           4.2000
    24           4.3000
    25           4.4000
    26           4.5000
    27           4.6000
    28           4.7000
    29           4.8000
    30           4.9000
    31           5.0000
 BLANK COMMON ARRAY WORKING SPACE=     15000

  FINITE DIFFERENCE MESH.  31 ROWS,    6 COLUMNS.  MESH SPACING. H=   0.0000, K=   1.3046

NRW1=  -3, NRW2= -3, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS SIMPLESUPRT.
BOUNDARY CONDITION AT LINE 2 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 3 IS  SYMMETRIC
BOUNDARY CONDITION AT LINE 4 IS  SYMMETRIC

LOAD A DATA

CARD COUNT =     1

 USER-LOAD FLAG =     0,  STARTING LOAD FACTOR =  1.00000000E+00,  LOAD STEP = -0.          ,  MAXIMUM LOAD =  1.00000000E+00

      PZ              PY             PX           JZ   JY   JX  ROW  COL
 0.             0.           .1.00000000E+00      0    6    2    1    0

 ISHIFT   ITERAT   SHIFT
   1        16     -0.

IPX=     4 IPY=     0 IPRD=     1 IPRS=     1 IPLOT=     -0

IWALL=     0,  NSTRI=     0,  NRINGS=     0,  IP=     3,  IN=    -0,  JN=    -0 .
```

Fig. 7-31 Excerpt of Output for Sample Case 10

```
CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=    2.418.  NR OF IO REQUESTS (TAPE2)=    14.     WORDS USED (TAPE2)=   45723.     WORDS TRANSFERRED (TAPE2)=       49819
 FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=    4.850.  NR OF IO REQUESTS (TAPE2)=    32.     WORDS USED (TAPE2)=   83385.     WORDS TRANSFERRED (TAPE2)=      129879
 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=    5.636.  NR OF IO REQUESTS (TAPE2)=    44.     WORDS USED (TAPE2)=  103120.     WORDS TRANSFERRED (TAPE2)=      198662

DETERMINANT OF STIFFNESS MATRIX=   1.2555249E+04*10.**  1218.    NUMBER OF NEGATIVE ROOTS =   0
      186 NODES,         792 EQUATIONS.   MAXIMUM BAND WIDTH =          54
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=    7.671.  NR OF IO REQUESTS (TAPE2)=    52.     WORDS USED (TAPE2)=  103120.     WORDS TRANSFERRED (TAPE2)=      248340

LINEAR SOLUTION,  PA=   1.0000000E+06,    PB=   0.
ROW 30,  X=       4.900, AXIAL LOAD=         -6.041545E-01,BENDING MOMENT=       0.

   ROW   1,  X=     0.0000
                                  OUTER SURFACE                                           INNER SURFACE
      X         Y       SIGMAX        SIGMAY           TAU           SIGMAX        SIGMAY           TAU      SIGMA(STRNGR)   SIGMA(RING)
    0.000     0.000   -3.3333E+01   -1.0001E+05    0.              -3.3333E+01   -1.0001E+05    0.
    0.000     1.385   -3.3333E+01   -1.0001E+05   -6.1918E-08      -3.3333E+01   -1.0001E+05   -6.4483E-08
    0.000     2.769   -3.3333E+01   -1.0001E+05   -4.3746E-08      -3.3333E+01   -1.0001E+05   -8.5444E-08
    0.000     4.154   -3.3333E+01   -1.0001E+05    3.5799E-08      -3.3333E+01   -1.0001E+05   -1.4673E-08
    0.000     5.538   -3.3333E+01   -1.0001E+05    6.4239E-08      -3.3333E+01   -1.0001E+05    2.0601E-08
    0.000     6.923   -3.3333E+01   -1.0001E+05    0.              -3.3333E+01   -1.0001E+05    0.

   ROW   4,  X=      .3000
                                  OUTER SURFACE                                           INNER SURFACE
      X         Y       SIGMAX        SIGMAY           TAU           SIGMAX        SIGMAY           TAU      SIGMA(STRNGR)   SIGMA(RING)
     .300     0.000    5.5381E+04   -3.5771E+03    0.              -5.5448E+04   -3.6826E+04    0.
     .300     1.385    5.5381E+04   -3.5771E+03   -5.9629E-08      -5.5448E+04   -3.6826E+04   -7.0523E-08
     .300     2.769    5.5381E+04   -3.5771E+03   -5.7830E-08      -5.5448E+04   -3.6826E+04   -8.9856E-08
     .300     4.154    5.5381E+04   -3.5771E+03    2.6566E-08      -5.5448E+04   -3.6826E+04   -1.7747E-08
     .300     5.538    5.5381E+04   -3.5771E+03    3.0947E-08      -5.5448E+04   -3.6826E+04    2.4054E-09
     .300     6.923    5.5381E+04   -3.5771E+03    0.              -5.5448E+04   -3.6826E+04    0.

   ROW  31,  X=     5.0000
                                  OUTER SURFACE                                           INNER SURFACE
      X         Y       SIGMAX        SIGMAY           TAU           SIGMAX        SIGMAY           TAU      SIGMA(STRNGR)   SIGMA(RING)
    5.000     0.000   -1.4318E+03    3.1900E+02    0.               1.3651E+03    1.1581E+03    0.
    5.000     1.385   -1.4318E+03    3.1900E+02    5.0639E-23       1.3651E+03    1.1581E+03    5.0336E-23
    5.000     2.769   -1.4318E+03    3.1900E+02    0.               1.3651E+03    1.1581E+03    0.
    5.000     4.154   -1.4318E+03    3.1900E+02    0.               1.3651E+03    1.1581E+03    0.
    5.000     5.538   -1.4318E+03    3.1900E+02    0.               1.3651E+03    1.1581E+03    0.
    5.000     6.923   -1.4318E+03    3.1900E+02    0.               1.3651E+03    1.1581E+03    0.
CP SECONDS=    9.941.  NR OF IO REQUESTS (TAPE2)=    85.     WORDS USED (TAPE2)=  104144.     WORDS TRANSFERRED (TAPE2)=      393279

   ROW   4,  X=      .3000
   COL        Y             W                 V                 U              BETAX            BETAY
     1     0.0000     3.915432E-02    0.                -4.071632E-02    8.226293E-02    0.
     2     1.3846     3.915432E-02   -1.174305E-16      -4.071632E-02    8.226293E-02   -2.697714E-14
     3     2.7692     3.915432E-02   -1.653779E-16      -4.071632E-02    8.226293E-02   -5.183654E-14
     4     4.1539     3.915432E-02   -2.517216E-16      -4.071632E-02    8.226293E-02   -6.531959E-14
     5     5.5385     3.915432E-02   -1.981931E-16      -4.071632E-02    8.226293E-02   -5.396161E-14
     6     6.9231     3.915432E-02    0.                -4.071632E-02    8.226293E-02    0.
```

Fig. 7-31 (Cont.)

ROW 14, X= 1.9000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	4.517744E-02	0.	-2.564316E-02	-2.358649E-03	0.
2	1.3846	4.517744E-02	2.661315E-17	-2.564316E-02	-2.358649E-03	1.065282E-14
3	2.7692	4.517744E-02	1.753163E-17	-2.564316E-02	-2.358649E-03	1.065463E-14
4	4.1539	4.517744E-02	4.677561E-18	-2.564316E-02	-2.358649E-03	-5.685277E-15
5	5.5385	4.517744E-02	-9.239995E-18	-2.564315E-02	-2.358649E-03	-1.563009E-14
6	6.9231	4.517744E-02	0.	-2.564316E-02	-2.358649E-03	0.

ROW 19, X= 3.4000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	4.150339E-02	0.	-1.263470E-02	-2.537533E-03	0.
2	1.3846	4.150339E-02	9.102901E-17	-1.263470E-02	-2.537533E-03	2.200662E-14
3	2.7692	4.150339E-02	1.424931E-16	-1.263470E-02	-2.537533E-03	3.052485E-14
4	4.1539	4.150339E-02	1.222217E-16	-1.263470E-02	-2.537533E-03	1.631604E-14
5	5.5385	4.150339E-02	6.677587E-17	-1.263470E-02	-2.537533E-03	-7.238979E-16
6	6.9231	4.150339E-02	0.	-1.263470E-02	-2.537533E-03	0.

ROW 31, X= 5.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	3.787426E-02	0.	4.336809E-19	0.	0.
2	1.3846	3.787426E-02	3.060636E-16	5.963617E-19	0.	3.630910E-14
3	2.7692	3.787426E-02	4.934659E-16	4.336809E-19	0.	6.171852E-14
4	4.1539	3.787426E-02	4.893835E-16	4.336809E-19	0.	6.314043E-14
5	5.5385	3.787426E-02	2.950624E-16	4.336809E-19	0.	3.617867E-14
6	6.9231	3.787426E-02	0.	4.336809E-19	0.	0.

ROW 4, X= .3000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.000000E+00	-6.060410E+02	0.	8.312155E+00	2.493646E+00	0.
2	1.3846	-1.000000E+00	-6.060410E+02	-1.952351E-09	8.312155E+00	2.493646E+00	8.173975E-13
3	2.7692	-1.000000E+00	-6.060410E+02	-2.215279E-09	8.312155E+00	2.493646E+00	2.401943E-12
4	4.1539	-1.000000E+00	-6.060410E+02	1.322925E-10	8.312155E+00	2.493646E+00	3.323487E-12
5	5.5385	-1.000000E+00	-6.060410E+02	6.262818E-10	8.312155E+00	2.493646E+00	2.710593E-12
6	6.9231	-1.000000E+00	-6.060410E+02	0.	8.312155E+00	2.493646E+00	0.

ROW 14, X= 1.9000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.000000E+00	-4.653459E+00	0.	-7.932087E-03	-2.379626E-03	0.
2	1.3846	-1.000000E+00	-4.653459E+00	-2.134919E-11	-7.932087E-03	-2.379626E-03	-3.840133E-13
3	2.7692	-1.000000E+00	-4.653459E+00	4.841339E-11	-7.932087E-03	-2.379626E-03	-4.341260E-13
4	4.1539	-1.000000E+00	-4.653459E+00	-5.854988E-11	-7.932087E-03	-2.379626E-03	-3.321333E-13
5	5.5385	-1.000000E+00	-4.653459E+00	-7.264472E-11	-7.932087E-03	-2.379626E-03	-7.654931E-14
6	6.9231	-1.000000E+00	-4.653459E+00	0.	-7.932087E-03	-2.379626E-03	0.

ROW 19, X= 3.4000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.000000E+00	-9.663914E-02	0.	3.800708E-03	1.140212E-03	0.
2	1.3846	-1.000000E+00	-9.663914E-02	-9.152151E-11	3.800708E-03	1.140212E-03	-5.110451E-13
3	2.7692	-1.000000E+00	-9.663914E-02	-1.026219E-11	3.800708E-03	1.140212E-03	-8.190597E-13
4	4.1539	-1.000000E+00	-9.663914E-02	-4.161621E-11	3.800708E-03	1.140212E-03	-8.946291E-13
5	5.5385	-1.000000E+00	-9.663914E-02	3.192355E-11	3.800708E-03	1.140212E-03	-5.875307E-13
6	6.9231	-1.000000E+00	-9.663914E-02	0.	3.800708E-03	1.140212E-03	0.

ROW 31, X= 5.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	-1.000000E+00	2.215581E+01	0.	-2.097643E-01	-6.292929E-02	0.
2	1.3846	-1.000000E+00	2.215581E+01	1.514613E-24	-2.097643E-01	-6.292929E-02	2.271919E-29
3	2.7692	-1.000000E+00	2.215581E+01	0.	-2.097643E-01	-6.292929E-02	0.
4	4.1539	-1.000000E+00	2.215581E+01	0.	-2.097643E-01	-6.292929E-02	0.
5	5.5385	-1.000000E+00	2.215581E+01	0.	-2.097643E-01	-6.292929E-02	0.
6	6.9231	-1.000000E+00	2.215581E+01	0.	-2.097643E-01	-6.292929E-02	0.

Fig. 7-31 (Cont.)

EIGENVALUE SHIFT= -0. . NUMBER OF NEGATIVE ROOTS= 0

ITERATION	EIGENVALUE (RAYLEIGH QUOTIENT)	EIGENVALUE (ACCELERATED ESTIMATE)
0	3.6721650E+01	2.2737368E-13
1	4.4343755E+00	1.9540659E+01
2	3.3564231E+00	3.8455990E+00
3	3.3197333E+00	-3.6638331E+00
4	3.3510192E+00	3.3492950E+00
5	3.2734072E+00	3.3287212E+00
6	3.2798400E+00	3.2793478E+00
7	3.2695053E+00	3.2758752E+00
8	3.2705680E+00	3.2704639E+00
9	3.2692036E+00	3.2699706E+00
10	3.2693690E+00	3.2693511E+00
11	3.2691883E+00	3.2692826E+00

THE BUCKLING LOAD BASED ON LINEAR BIFURCATION THEORY IS 3.269283E+00 TIMES THE STARTING LOAD.

ROW 4, X= .3000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	9.946699E-01	0.	-4.772973E-03	-4.923343E-01	0.
2	1.3846	6.060373E-01	-1.910220E-02	-3.859737E-03	-4.037708E-01	-2.827907E+00
3	2.7692	3.103495E-01	-3.095241E-02	-1.471174E-03	-1.636080E-01	-4.595591E+00
4	4.1539	-3.060373E-01	-3.100730E-02	1.476610E-03	1.472341E-01	-4.620240E+00
5	5.5385	-8.076344E-01	-1.919101E-02	3.857660E-03	4.100251E-01	-2.867790E+00
6	6.9231	-1.000000E+00	0.	4.766254E-03	5.130735E-01	0.

ROW 14, X= 1.9000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	3.853322E-04	0.	1.375787E-04	-4.739751E-03	0.
2	1.3846	3.117420E-04	4.107060E-05	1.113346E-04	-3.833784E-03	-1.109970E-03
3	2.7692	1.190780E-04	6.645613E-05	4.251647E-05	-1.464475E-03	-1.796008E-03
4	4.1539	-1.190725E-04	6.645922E-05	-4.251314E-05	1.464391E-03	-1.796040E-03
5	5.5385	-3.117441E-04	4.107560E-05	-1.113058E-04	3.833650E-03	-1.110030E-03
6	6.9231	-3.853390E-04	0.	-1.375529E-04	4.739966E-03	0.

ROW 19, X= 3.4000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-2.429973E-06	0.	-1.064808E-07	1.068629E-05	0.
2	1.3846	-1.965377E-06	4.453342E-08	-8.614776E-08	8.645410E-06	6.936272E-06
3	2.7692	-7.505563E-07	7.212563E-08	-3.291111E-08	3.302285E-06	1.122307E-05
4	4.1539	7.509973E-07	7.212368E-08	3.290134E-08	-3.302229E-06	1.122301E-05
5	5.5385	1.965073E-06	4.457241E-08	8.615148E-08	-8.645431E-06	6.936172E-06
6	6.9231	2.428959E-06	0.	1.064923E-07	-1.068636E-05	0.

ROW 31, X= 5.0000

COL	Y	W	V	U	BETAX	BETAY
1	0.0000	-3.259796E-11	0.	2.867952E-25	0.	0.
2	1.3846	-2.637269E-11	-2.341546E-10	5.169579E-25	0.	1.400325E-10
3	2.7692	-1.007472E-11	-3.788701E-10	8.828596E-26	0.	2.265768E-10
4	4.1539	1.007358E-11	-3.788701E-10	-1.033976E-25	0.	2.265762E-10
5	5.5385	2.636941E-11	-2.341546E-10	0.	0.	1.400315E-10
6	6.9231	3.259360E-11	0.	0.	0.	0.

CP SECONDS= 15.432. NR OF IO REQUESTS (TAPE2)= 223. WORDS USED (TAPE2)= 124983. WORDS TRANSFERRED (TAPE2)= 1283697

Fig. 7-31 (Cont.)

7.11 SAMPLE CASE 11 – PLASTICITY

The stress distribution in the interior of a square plate subjected to concentrated shear force at one edge is to be determined using the plasticity branch of the STAGS computer program. The geometry and boundary conditions are shown in Fig. 7-32.

The stress-strain curve is described in the input data utilizing five distinct points (material components). The equivalent White-Besseling material components are shown in the output. Since the stresses are uniform across the thickness in this problem, only three points are used for the numerical integration through the thickness of the plate.

Points at which the effective stress is in the inelastic range are designated with an asterisk as shown in the output. To continue a nonlinear plasticity run, only the last record can be used for a restart (ISTART=3), because the strains for records 1 and 2 are not saved.

The input cards associated with this case are displayed in Fig. 7-33. Portions of the output are presented in Fig. 7-34.

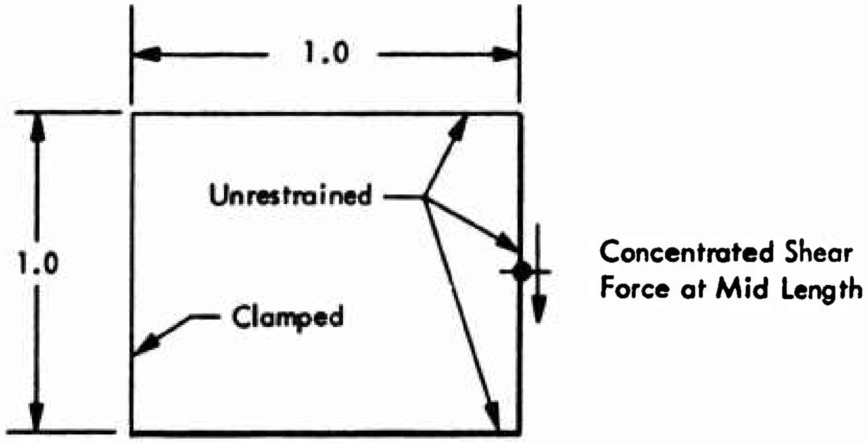


Fig. 7-32 Sample Case 11 – Plasticity (Plate)

SAMPLE CASE 11
INPUT

```
   SAMPLE CASE 11 - PLASTICITY                                                    C-1
     3         3        1                                                         G-1
1.             1.                                                                 G-2
   -7    -7                                                                       D-1
0.             0.05        0.25        0.5         0.75        0.95        1.0    D-8
0.             0.05        0.25        0.5         0.75        0.95        1.     D-9
10000000.      0.3         0.05        3.                                         I-1
     3         5                                                                  I-2
25000.         37500.      45000.      49000.      50000.                         I-3
0.0025         0.0044      0.0078      0.0113      0.0140                         I-4
     2         3        3        3                                                B-1
     1         0 1.              .5          3.                                   L-1
0.             100.        0.               0       1       0       7       4     L-2
                                    0     60        4       2       1             P-1B
     1         1        1                                                         O-1
```

Fig. 7-33 Display of Input Card for Sample Case 11

SAMPLE CASE 11 – OUTPUT

```
  SAMPLE CASE 11 - PLASTICITY
PLASTICITY NON-LINEAR ANALYSIS.          1 LOAD PATTERNS.

TYPE OF SURFACE IS     PLATE

SURFACE CONSTANTS =  1.0000000E+00, 1.0000000E+00,

E =  1.0000000E+07   NU =  3.0000000E-01    T =  5.0000000E-02    AK**2 =  3.0000000E+00

NL =  3  IC =  5

STRESS-STRAIN CURVE          S              E
                     2.5000000E+04 2.5000000E-03
                     3.7500000E+04 4.4000000E-03
                     4.5000000E+04 7.8000000E-03
                     4.9000000E+04 1.1300000E-02
                     5.0000000E+04 1.4800000E-02

EFFECTIVE MATERIAL COMPONENTS BASED ON THE WHITE-BESSELING FORMULATION
                               YY             VV
                     2.5000000E+04 3.7500000E-01
                     4.5000000E+04 4.2803030E-01
                     8.3076923E+04 9.6389426E-02
                     1.2294615E+05 6.8322206E-02
                     1.5384615E+05 3.2259065E-02

 BLANK COMMON ARRAY WORKING SPACE=     15000

  FINITE DIFFERENCE MESH.   7 ROWS,    7 COLUMNS.  MESH SPACING. H=    0.0000, K=   0.0000

NRA1=  -0, NRA2= -0, NCL1=  -0
BOUNDARY CONDITION AT LINE 1 IS   CLAMPED
BOUNDARY CONDITION AT LINE 2 IS UNRESTRAINED
BOUNDARY CONDITION AT LINE 3 IS UNRESTRAINED
BOUNDARY CONDITION AT LINE 4 IS UNRESTRAINED

LOAD & DATA

CARD COUNT =     1

 USER-LOAD FLAG =      0.  STARTING LOAD FACTOR =  1.0000000E+00,  LOAD STEP =  5.0000000E-01,  MAXIMUM LOAD =  3.0000000E+00

      PZ               PY              PX           JZ   JY   JX  ROW  COL
 0.           1.0000000E+02 0.                      0    1    0    7    4

ERROR TOLERANCE =    1.0000000E-04 UNDERRELAXATION =    -0.
 ISTART   ISEC   ICUT  INENT  ISTRAT
    0      60     4      2      1

IPX=    -0 IPY=    -0 IPRD=    -0 IPRS=    -0 IPLOT=    -0


THE FOLLOWING STIFFNESS COEFFICIENTS ARE CALCULATED IN SUBROUTINE CF02
     CCC(I,1)           CCC(I,2)           CCC(I,3)           CCC(I,4)           CCC(I,5)           CCC(I,6)

     5.494505E+05       0.                 0.                 0.                 0.                 0.
     1.648352E+05       5.494505E+05       0.                 0.                 0.                 0.
     0.                 0.                 1.923077E+05       0.                 0.                 0.
     0.                 0.                 0.                 1.144689E+02       0.                 0.
     0.                 0.                 0.                 3.434066E+01       1.144689E+02       0.
     0.                 0.                 0.                 0.                                    4.006410E+01
```

Fig. 7-34 Excerpt of Output for [illegible] 11

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 CALCULATION OF FINITE DIFFERENCE FORMULAS AND GEOMETRIC CONSTANTS COMPLETED.
CP SECONDS=      .786.  NR OF IO REQUESTS (TAPE2)=     9.      WORDS USED (TAPE2)=   14894.     WORDS TRANSFERRED (TAPE2)=       16942

 FORMATION OF STIFFNESS MATRICES FOR ALL SUBREGIONS COMPLETED.
CP SECONDS=     1.544.  NR OF IO REQUESTS (TAPE2)=    19.      WORDS USED (TAPE2)=   28764.     WORDS TRANSFERRED (TAPE2)=       46738

 ASSEMBLY OF TOTAL STIFFNESS MATRIX COMPLETED.
CP SECONDS=     1.775.  NR OF IO REQUESTS (TAPE2)=    23.      WORDS USED (TAPE2)=   35699.     WORDS TRANSFERRED (TAPE2)=       67535

DISPLACEMENT COMPONENT V       X=     1.0250E+00       Y=     1.0250E+00,   HAS BEEN ELIMINATED
  THE INITIAL MAIN DIAGONAL VALUE =  4.6359890E+04,   THE FINAL VALUE =  3.7020072E-07
DISPLACEMENT COMPONENT U       X=     1.0250E+00       Y=     1.0250E+00,   HAS BEEN ELIMINATED
  THE INITIAL MAIN DIAGONAL VALUE =  4.6359890E+04,   THE FINAL VALUE =  6.9104135E-07

DETERMINANT OF STIFFNESS MATRIX=   1.1704462E+06*10.**   448.   NUMBER OF NEGATIVE ROOTS =   0
         49 NODES,       243 EQUATIONS.   MAXIMUM BAND WIDTH =          60
 MATRIX DECOMPOSITION COMPLETED.
CP SECONDS=     2.507.  NR OF IO REQUESTS (TAPE2)=    27.      WORDS USED (TAPE2)=   35699.     WORDS TRANSFERRED (TAPE2)=       82429


LINEAR SOLUTION.  FA=    1.000000E+00,   PB=    0.



                                       ROW   1,  X=     0.0000
   COL       THETA           W                  V                   U
    1       0.0000     0.                0.                 0.
    2        .0500     0.                0.                 0.
    3        .2500     0.                0.                 0.
    4        .5000     0.                0.                 0.
    5        .7500     0.                0.                 0.
    6        .9500     0.                0.                 0.
    7       1.0000     0.                0.                 0.


                                       ROW   7,  X=     1.0000
   COL       THETA           W                  V                   U
    1       0.0000     0.                1.31216E-03      5.99990E-04
    2        .0500     0.                1.30666E-03      4.92953E-04
    3        .2500     0.                1.37427E-03      2.90999E-04
    4        .5000     0.                1.44758E-03      6.59195E-17
    5        .7500     0.                1.37427E-03     -2.90999E-04
    6        .9500     0.                1.30666E-03     -4.92953E-04
    7       1.0000     0.                1.31216E-03     -5.99980E-04
CP SECONDS=     2.998.  NR OF IO REQUESTS (TAPE2)=    38.      WORDS USED (TAPE2)=   42957.     WORDS TRANSFERRED (TAPE2)=      113352

STRESS AND MOMENT RESULTANTS COMPUTED ASSUMING TOTAL STRAINS ARE ELASTIC

  ROW    1,  X=     0.0000
  COL        Y             NX                NY                NXY                MX             MY              MXY
   1       0.0000     8.683755E+02     2.605127E+02     1.589834E+02     0.               0.              0.
   2        .0500     4.391582E+02     1.494505E+02     1.210192E+02     0.               0.              0.
   3        .2500     2.284633E+02     6.853898E+01     8.690503E+01     0.               0.              0.
   4        .5000     9.783378E-10     3.004312E-10     9.075507E+01     0.               0.              0.
   5        .7500    -2.284633E+02    -6.853898E+01     8.690503E+01     0.               0.              0.
   6        .9500    -4.581582E+02    -1.494505E+02     1.210192E+02     0.               0.              0.
   7       1.0000    -8.683755E+02    -2.605127E+02     1.589834E+02     0.               0.              0.
```

Fig. 7-34 (Cont.)

```
ROW   7,  X=     1.0000
COL       Y            NX              NY              NXY            MX              MY              MXY
  1    0.0000    5.815027E+00    5.815027E+00    5.815027E+00    0.              0.              0.
  2     .0500   -1.109473E+01   -4.898947E+01   -2.192180E+01    0.              0.              0.
  3     .2500    3.273791E+01    3.356668E+02    9.765794E+01    0.              0.              0.
  4     .5000   -3.363814E-09   -2.507152E-09    2.449745E+02    0.              0.              0.
  5     .7500   -3.273791E+01   -3.356668E+02    9.765794E+01    0.              0.              0.
  6     .9500    1.109473E+01    4.898947E+01   -2.192180E+01    0.              0.              0.
  7    1.0000   -5.815027E+00   -5.815027E+00    5.815027E+00    0.              0.              0.
ROW  5,  X=      .550,  AXIAL LOAD=       4.639753E-10,BENDING MOMENT=     0.
CP SECONDS=    3.236,  NR OF IO REQUESTS (TAPE2)=    40.     WORDS USED (TAPE2)=   42957.    WORDS TRANSFERRED (TAPE2)=      126410
CP SECONDS=    3.240,  NR OF IO REQUESTS (TAPE2)=    40.     WORDS USED (TAPE2)=   42957.    WORDS TRANSFERRED (TAPE2)=      126410

BEGIN NON-LINEAR ITERATION FOR LOAD STEP    1,  PA= 1.0000E+00,  PB= 0.

ITERATION  MAXIMUM DISPLACEMENT CHANGE     COMPONENT     ROW    COL       REL ERROR    RELAXATION FACTOR
    1              -3.707420E-14              V           7      8      3.062427E-11              1.0000

     X              Y              Z           SIGMA1       SIGMA2         TAU          EPS1          EPS2          EPS12

    0.          0.          -2.5000E-02   1.7368E+04   5.2103E+03   3.1797E+03   1.5834E-03   0.            8.2671E-04
    0.          0.           1.1102E-16   1.7368E+04   5.2103E+03   3.1797E+03   1.5834E-03   0.            8.2671E-04
    0.          0.           2.5000E-02   1.7368E+04   5.2103E+03   3.1797E+03   1.5834E-03   0.            8.2671E-04
    0.          5.0000E-02  -2.5000E-02   9.9634E+03   2.9890E+03   2.4204E+03   9.0667E-04   2.7756E-17    6.2930E-04
    0.          5.0000E-02   1.1102E-16   9.9634E+03   2.9890E+03   2.4204E+03   9.0667E-04   2.7756E-17    6.2930E-04
    0.          5.0000E-02   2.5000E-02   9.9634E+03   2.9890E+03   2.4204E+03   9.0667E-04   2.7756E-17    6.2930E-04
    0.          2.5000E-01  -2.5000E-02   4.5693E+03   1.3708E+03   1.7381E+03   4.1580E-04  -1.3878E-17    4.5191E-04
    0.          2.5000E-01   1.1102E-16   4.5693E+03   1.3708E+03   1.7381E+03   4.1580E-04  -1.3878E-17    4.5191E-04
    0.          2.5000E-01   2.5000E-02   4.5693E+03   1.3708E+03   1.7381E+03   4.1580E-04  -1.3878E-17    4.5191E-04
    0.          5.0000E-01  -2.5000E-02   4.5751E-11   1.5250E-10   1.8151E+03   1.3458E-32   1.3878E-17    4.7193E-04
    0.          5.0000E-01   1.1102E-16   4.5751E-11   1.5250E-10   1.8151E+03  -5.9767E-47   1.3878E-17    4.7193E-04
    0.          5.0000E-01   2.5000E-02   4.5751E-11   1.5250E-10   1.8151E+03  -1.3458E-32   1.3878E-17    4.7193E-04
    0.          7.5000E-01  -2.5000E-02  -4.5693E+03  -1.3708E+03   1.7381E+03  -4.1580E-04  -1.3878E-17    4.5191E-04
    0.          7.5000E-01   1.1102E-16  -4.5693E+03  -1.3708E+03   1.7381E+03  -4.1580E-04  -1.3878E-17    4.5191E-04
    0.          7.5000E-01   2.5000E-02  -4.5693E+03  -1.3708E+03   1.7381E+03  -4.1580E-04  -1.3878E-17    4.5191E-04
    0.          9.5000E-01  -2.5000E-02  -9.9634E+03  -2.9890E+03   2.4204E+03  -9.0667E-04   0.            6.2930E-04
    0.          9.5000E-01   1.1102E-16  -9.9634E+03  -2.9890E+03   2.4204E+03  -9.0667E-04   0.            6.2930E-04
    0.          9.5000E-01   2.5000E-02  -9.9634E+03  -2.9890E+03   2.4204E+03  -9.0667E-04   0.            6.2930E-04
    0.          1.0000E+00  -2.5000E-02  -1.7368E+04  -5.2103E+03   3.1797E+03  -1.5834E-03   0.            8.2671E-04
    0.          1.0000E+00   1.1102E-16  -1.7368E+04  -5.2103E+03   3.1797E+03  -1.5834E-03   0.            8.2671E-04
    0.          1.0000E+00   2.5000E-02  -1.7368E+04  -5.2103E+03   3.1797E+03  -1.5834E-03   0.            8.2671E-04

    1.0000E+00  0.          -2.5000E-02   1.1630E+02   1.1630E+02   1.1630E+02   8.1410E-06   8.1410E-06    3.0238E-05
    1.0000E+00  0.           1.1102E-16   1.1630E+02   1.1630E+02   1.1630E+02   8.1410E-06   8.1410E-06    3.0238E-05
    1.0000E+00  0.           2.5000E-02   1.1630E+02   1.1630E+02   1.1630E+02   8.1410E-06   8.1410E-06    3.0238E-05
    1.0000E+00  5.0000E-02  -2.5000E-02  -2.2189E+02  -9.7979E+02  -4.3844E+02   7.2042E-06  -9.1322E-05   -1.1399E-04
    1.0000E+00  5.0000E-02   1.1102E-16  -2.2189E+02  -9.7979E+02  -4.3844E+02   7.2042E-06  -9.1322E-05   -1.1399E-04
    1.0000E+00  5.0000E-02   2.5000E-02  -2.2189E+02  -9.7979E+02  -4.3844E+02   7.2042E-06  -9.1322E-05   -1.1399E-04
    1.0000E+00  2.5000E-01  -2.5000E-02   6.5474E+02   6.7133E+03   1.9532E+03  -1.3593E-04   6.5169E-04    5.0782E-04
    1.0000E+00  2.5000E-01   1.1102E-16   6.5474E+02   6.7133E+03   1.9532E+03  -1.3593E-04   6.5169E-04    5.0782E-04
    1.0000E+00  2.5000E-01   2.5000E-02   6.5474E+02   6.7133E+03   1.9532E+03  -1.3593E-04   6.5169E-04    5.0782E-04
    1.0000E+00  5.0000E-01  -2.5000E-02   9.1502E-11   3.0501E-10   4.8995E+03   1.3801E-33   2.7756E-17    1.2739E-03
    1.0000E+00  5.0000E-01   1.1102E-16   9.1502E-11   3.0501E-10   4.8995E+03  -6.1290E-48   2.7756E-17    1.2739E-03
    1.0000E+00  5.0000E-01   2.5000E-02   9.1502E-11   3.0501E-10   4.8995E+03  -1.3801E-33   2.7756E-17    1.2739E-03
    1.0000E+00  7.5000E-01  -2.5000E-02  -6.5474E+02  -6.7133E+03   1.9532E+03   1.3593E-04  -6.5169E-04    5.0782E-04
    1.0000E+00  7.5000E-01   1.1102E-16  -6.5474E+02  -6.7133E+03   1.9532E+03   1.3593E-04  -6.5169E-04    5.0782E-04
    1.0000E+00  7.5000E-01   2.5000E-02  -6.5474E+02  -6.7133E+03   1.9532E+03   1.3593E-04  -6.5169E-04    5.0782E-04
    1.0000E+00  9.5000E-01  -2.5000E-02   2.2189E+02   9.7979E+02  -4.3844E+02  -7.2042E-06   9.1322E-05   -1.1399E-04
    1.0000E+00  9.5000E-01   1.1102E-16   2.2189E+02   9.7979E+02  -4.3844E+02  -7.2042E-06   9.1322E-05   -1.1399E-04
    1.0000E+00  9.5000E-01   2.5000E-02   2.2189E+02   9.7979E+02  -4.3844E+02  -7.2042E-06   9.1322E-05   -1.1399E-04
    1.0000E+00  1.0000E+00  -2.5000E-02  -1.1630E+02  -1.1630E+02   1.1630E+02  -8.1410E-06  -8.1410E-06    3.0238E-05
    1.0000E+00  1.0000E+00   1.1102E-16  -1.1630E+02  -1.1630E+02   1.1630E+02  -8.1410E-06  -8.1410E-06    3.0238E-05
    1.0000E+00  1.0000E+00   2.5000E-02  -1.1630E+02  -1.1630E+02   1.1630E+02  -8.1410E-06  -8.1410E-06    3.0238E-05
AXIAL LOAD ADJUSTED FOR PLASTIC STRAIN = 4.8805340E-12
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Fig. 7-34 (Cont.)

```
LOAD STEP  1.  PA=1.0000E+00.  PB=0.        .    2 ITERATIONS.  RELATIVE ERROR=     3.062427E-11
CP SECONDS=    4.350.  NR OF IO REQUESTS (TAPE2)=    54.      WORDS USED (TAPE2)=  42957.     WORDS TRANSFERRED (TAPE2)=      189605
```

ROW 1, X= 0.0000

COL	THETA	W	V	U
1	0.0000	0.	0.	0.
2	.0500	0.	0.	0.
3	.2500	0.	0.	0.
4	.5000	0.	0.	0.
5	.7500	0.	0.	0.
6	.9500	0.	0.	0.
7	1.0000	0.	0.	0.

ROW 7, X= 1.0000

COL	THETA	W	V	U
1	0.0000	1.13907E-30	1.31216E-03	5.99980E-04
2	.0500	1.04363E-30	1.30666E-03	4.92953E-04
3	.2500	6.22056E-31	1.37427E-03	2.90999E-04
4	.5000	-1.76465E-32	1.44758E-03	1.30134E-18
5	.7500	-6.70494E-31	1.37427E-03	-2.90999E-04
6	.9500	-1.97477E-30	1.30666E-03	-4.92953E-04
7	1.0000	-1.16588E-30	1.31216E-03	-5.99980E-04

STRESS AND MOMENT RESULTANTS COMPUTED ASSUMING TOTAL STRAINS ARE ELASTIC

ROW 1, X= 0.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	6.693755E+02	2.605127E+02	1.589834E+02	2.453496E-28	7.360488E-29	4.491341E-43
2	.0500	4.981652E+02	1.494505E+02	1.210192E+02	-6.037768E-29	-1.011330E-29	2.507088E-44
3	.2500	2.286633E+02	6.853098E+01	8.690503E+01	-1.438379E-28	-4.315137E-29	1.754430E-44
4	.5000	2.267547E-12	7.625158E-12	9.075507E+01	-6.162204E-29	-1.448661E-29	-1.403544E-44
5	.7500	-2.286633E+02	-6.853098E+01	8.690503E+01	2.770912E-28	8.312737E-29	-8.421265E-44
6	.9500	-4.981652E+02	-1.494505E+02	1.210192E+02	8.160539E-28	2.450562E-28	0.
7	1.0000	-6.693755E+02	-2.605127E+02	1.589834E+02	6.326070E-28	1.097821E-28	4.491341E-43

ROW 7, X= 1.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	5.815027E+00	5.815027E+00	5.815027E+00	1.350446E-28	1.411445E-28	1.051056E-29
2	.0500	-1.109473E+01	-4.898947E+01	-2.192100E+01	-4.670091E-29	1.516967E-28	1.477456E-29
3	.2500	3.273701E+01	3.356685E+02	9.765794E+01	7.544354E-29	2.303777E-28	2.125043E-28
4	.5000	4.575095E-12	1.525332E-11	2.449745E+02	1.529725E-30	2.425743E-29	2.964572E-28
5	.7500	-3.273701E+01	-3.356659E+02	9.765794E+01	-7.401726E-29	-2.963948E-28	2.298383E-28
6	.9500	1.109473E+01	4.898347E+01	-2.192100E+01	4.942502E-29	-1.511960E-28	2.995910E-29
7	1.0000	-5.815027E+00	-5.815027E+00	5.815027E+00	-1.663129E-28	-1.510703E-28	1.051056E-29

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ROW  6.  X=       .950,  AXIAL LOAD=       4.982983E-12,BENDING MOMENT=      0.
```

Fig. 7-34 (Cont.)

BEGIN NON-LINEAR ITERATION FOR LOAD STEP 8, PA= 3.0000E+00, PB= 0.

ITERATION	MAXIMUM DISPLACEMENT CHANGE	COMPONENT	ROW	COL	REL ERROR	RELAXATION FACTOR
0	9.539456E-06	V	6	7	1.096538E-03	1.0500
1	6.007900E-06	V	7	0	6.846538E-04	1.0500
2	2.999513E-06	V	7	0	3.429075E-04	1.0500
3	1.647934E-06	V	7	0	1.878576E-04	1.0500
4	9.674243E-07	V	7	0	1.102973E-04	1.0500
5	5.859463E-07	V	7	0	6.692723E-05	1.0500

X	Y	Z	SIGMA1	SIGMA2	TAU	EPS1	EPS2	EPS12
0.	0.	-2.5000E-02	4.4452E+04*	1.6777E+04*	7.9740E+03*	6.1690E-03*	-2.2204E-16*	3.5092E-03*
0.	0.	1.1102E-16	4.4452E+04*	1.6777E+04*	7.9740E+03*	6.1690E-03*	-2.2204E-16*	3.5042E-03*
0.	0.	2.5000E-02	4.4452E+04*	1.6777E+04*	7.9740E+03*	6.1690E-03*	-2.2204E-16*	3.5042E-03*
0.	5.0000E-02	-2.5000E-02	3.0874E+04*	9.8248E+03*	6.7900E+03*	3.0557E-03*	1.1102E-16*	1.9731E-03*
0.	5.0000E-02	1.1102E-16	3.0874E+04*	9.8248E+03*	6.7900E+03*	3.0557E-03*	1.1102E-16*	1.9731E-03*
0.	5.0000E-02	2.5000E-02	3.0874E+04*	9.8248E+03*	6.7900E+03*	3.0557E-03*	1.1102E-16*	1.9731E-03*
0.	2.5000E-01	-2.5000E-02	1.4561E+04	4.3683E+03	5.5882E+03	1.3251E-03	-1.1102E-16	1.4529E-03
0.	2.5000E-01	1.1102E-16	1.4561E+04	4.3683E+03	5.5882E+03	1.3251E-03	-1.1102E-16	1.4529E-03
0.	2.5000E-01	2.5000E-02	1.4561E+04	4.3683E+03	5.5882E+03	1.3251E-03	-1.1102E-16	1.4529E-03
0.	5.0000E-01	-2.5000E-02	-4.7103E-09	-8.5794E-10	5.5592E+03	-4.4529E-16	5.5511E-17	1.4454E-03
0.	5.0000E-01	1.1102E-16	-4.6971E-09	-9.5402E-10	5.5592E+03	-4.4409E-16	5.5511E-17	1.4454E-03
0.	5.0000E-01	2.5000E-02	-4.6839E-09	-9.5005E-10	5.5592E+03	-4.4289E-16	5.5511E-17	1.4454E-03
0.	7.5000E-01	-2.5000E-02	-1.4561E+04	-4.3683E+03	5.5882E+03	-1.3251E-03	0.	1.4529E-03
0.	7.5000E-01	1.1102E-16	-1.4561E+04	-4.3683E+03	5.5882E+03	-1.3251E-03	0.	1.4529E-03
0.	7.5000E-01	2.5000E-02	-1.4561E+04	-4.3683E+03	5.5882E+03	-1.3251E-03	0.	1.4529E-03
0.	9.5000E-01	-2.5000E-02	-3.0874E+04*	-9.8248E+03*	6.7900E+03*	-3.0557E-03*	1.1102E-16*	1.9731E-03*
0.	9.5000E-01	1.1102E-16	-3.0874E+04*	-9.8248E+03*	6.7900E+03*	-3.0557E-03*	1.1102E-16*	1.9731E-03*
0.	9.5000E-01	2.5000E-02	-3.0874E+04*	-9.8248E+03*	6.7900E+03*	-3.0557E-03*	1.1102E-16*	1.9731E-03*
0.	1.0000E+00	-2.5000E-02	-4.4452E+04*	-1.6777E+04*	7.9740E+03*	-6.1690E-03*	0. *	3.5032E-03*
0.	1.0000E+00	1.1102E-16	-4.4452E+04*	-1.6777E+04*	7.9740E+03*	-6.1690E-03*	0. *	3.5042E-03*
0.	1.0000E+00	2.5000E-02	-4.4452E+04*	-1.6777E+04*	7.9740E+03*	-6.1690E-03*	0. *	3.5032E-03*
1.0000E+00	0.	-2.5000E-02	2.0735E+02	2.0735E+02	2.0735E+02	1.4515E-05	1.4515E-05	5.3912E-05
1.0000E+00	0.	1.1102E-16	2.0735E+02	2.0735E+02	2.0735E+02	1.4515E-05	1.4515E-05	5.3912E-05
1.0000E+00	0.	2.5000E-02	2.0735E+02	2.0735E+02	2.0735E+02	1.4515E-05	1.4515E-05	5.3912E-05
1.0000E+00	5.0000E-02	-2.5000E-02	-6.0886E+02	-3.0796E+03	-1.3148E+03	3.1503E-05	-2.8970E-04	-3.4185E-04
1.0000E+00	5.0000E-02	1.1102E-16	-6.0886E+02	-3.0796E+03	-1.3148E+03	3.1503E-05	-2.8970E-04	-3.4185E-04
1.0000E+00	5.0000E-02	2.5000E-02	-6.0886E+02	-3.0796E+03	-1.3148E+03	3.1503E-05	-2.8970E-04	-3.4185E-04
1.0000E+00	2.5000E-01	-2.5000E-02	1.9425E+03	2.0203E+04	5.9043E+03	-4.1183E-04	1.9620E-03	1.5351E-03
1.0000E+00	2.5000E-01	1.1102E-16	1.9425E+03	2.0203E+04	5.9043E+03	-4.1183E-04	1.9620E-03	1.5351E-03
1.0000E+00	2.5000E-01	2.5000E-02	1.9425E+03	2.0203E+04	5.9043E+03	-4.1183E-04	1.9620E-03	1.5351E-03
1.0000E+00	5.0000E-01	-2.5000E-02	1.2207E-09*	3.6359E-10*	1.4645E+04*	1.1135E-16*	-1.0003E-18*	3.8438E-03*
1.0000E+00	5.0000E-01	1.1102E-16	1.2206E-09*	3.7448E-10*	1.4645E+04*	1.1102E-16*	5.5589E-33*	3.8438E-03*
1.0000E+00	5.0000E-01	2.5000E-02	1.2206E-09*	3.8537E-10*	1.4645E+04*	1.1070E-16*	1.0003E-18*	3.8438E-03*
1.0000E+00	7.5000E-01	-2.5000E-02	-1.9425E+03	-2.0203E+04	5.9043E+03	4.1183E-04	-1.9620E-03	1.5351E-03
1.0000E+00	7.5000E-01	1.1102E-16	-1.9425E+03	-2.0203E+04	5.9043E+03	4.1183E-04	-1.9620E-03	1.5351E-03
1.0000E+00	7.5000E-01	2.5000E-02	-1.9425E+03	-2.0203E+04	5.9043E+03	4.1183E-04	-1.9620E-03	1.5351E-03
1.0000E+00	9.5000E-01	-2.5000E-02	6.0886E+02	3.0796E+03	-1.3148E+03	-3.1503E-05	2.8970E-04	-3.4185E-04
1.0000E+00	9.5000E-01	1.1102E-16	6.0886E+02	3.0796E+03	-1.3148E+03	-3.1503E-05	2.8970E-04	-3.4185E-04
1.0000E+00	9.5000E-01	2.5000E-02	6.0886E+02	3.0796E+03	-1.3148E+03	-3.1503E-05	2.8970E-04	-3.4185E-04
1.0000E+00	1.0000E+00	-2.5000E-02	-2.0735E+02	-2.0735E+02	2.0735E+02	-1.4515E-05	-1.4515E-05	5.3912E-05
1.0000E+00	1.0000E+00	1.1102E-16	-2.0735E+02	-2.0735E+02	2.0735E+02	-1.4515E-05	-1.4515E-05	5.3912E-05
1.0000E+00	1.0000E+00	2.5000E-02	-2.0735E+02	-2.0735E+02	2.0735E+02	-1.4515E-05	-1.4515E-05	5.3912E-05

AXIAL LOAD ADJUSTED FOR PLASTIC STRAIN = 1.0516032E-11

Fig. 7-34 (Cont.)

LOAD STEP 6, PA=3.0000E+00, PB=0. . 6 ITERATIONS. RELATIVE ERROR= 6.692723E-05
CP SECONDS= 26.011. NR OF IO REQUESTS (TAPE2)= 404. WORDS USED (TAPE2)= 44493. WORDS TRANSFERRED (TAPE2)= 1634002

MAXIMUM LOAD VALUE ATTAINED

NONLINEAR COLLAPSE ANALYSIS DISCONTINUED

ROW 1, X= 0.0000

COL	THETA	W	V	U
1	0.0000	0.	0.	0.
2	.0500	0.	0.	0.
3	.2500	0.	0.	0.
4	.5000	0.	0.	0.
5	.7500	0.	0.	3.46945E-18
6	.9500	0.	0.	1.73472E-18
7	1.0000	0.	0.	0.

ROW 7, X= 1.0000

COL	THETA	W	V	U
1	0.0000	-2.35018E-17	4.03971E-03	1.84853E-03
2	.0500	-2.13362E-17	4.02197E-03	1.51832E-03
3	.2500	-1.23565E-17	4.22459E-03	8.95731E-04
4	.5000	-1.25911E-18	4.44531E-03	-5.67362E-18
5	.7500	7.14766E-18	4.22459E-03	-8.95731E-04
6	.9500	2.22923E-17	4.02197E-03	-1.51832E-03
7	1.0000	2.38229E-17	4.03971E-03	-1.84853E-03

STRESS AND MOMENT RESULTANTS COMPUTED ASSUMING TOTAL STRAINS ARE ELASTIC

ROW 1, X= 0.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	3.389535E+03	1.016961E+03	6.746613E+02	1.203559E-13	3.610677E-14	0.
2	.0500	1.578348E+03	5.036843E+02	3.794439E+02	5.657351E-14	1.697205E-14	0.
3	.2500	7.280523E+02	2.184157E+02	2.794102E+02	-6.644991E-15	-1.993497E-15	1.148151E-29
4	.5000	-2.348549E-10	-4.270089E-11	2.779602E+02	5.505959E-15	1.651788E-15	1.481485E-30
5	.7500	-7.280523E+02	-2.184157E+02	2.794102E+02	-1.704883E-14	-5.114550E-15	1.293953E-29
6	.9500	-1.678948E+03	-5.036943E+02	3.794489E+02	-3.022704E-14	-9.068112E-15	3.160500E-29
7	1.0000	-3.389535E+03	-1.016961E+03	6.746613E+02	-1.869723E-13	-5.609168E-14	6.321001E-29

ROW 7, X= 1.0000

COL	Y	NX	NY	NXY	MX	MY	MXY
1	0.0000	1.036756E+01	1.036766E+01	1.036766E+01	2.265654E-15	-9.149860E-16	1.911254E-16
2	.0500	-3.044303E+01	-1.539821E+02	-6.573991E+01	1.343197E-15	-1.917763E-15	1.886016E-15
3	.2500	4.712511E+01	1.010138E+03	2.952155E+02	-1.723092E-14	-4.859939E-15	-4.890893E-15
4	.5000	6.100127E-11	1.830038E-11	7.386190E+02	-1.188521E-17	4.497533E-15	-1.301700E-15
5	.7500	-9.712511E+01	-1.010138E+03	2.952155E+02	-1.185188E-14	-2.302120E-14	-7.877809E-15
6	.9500	3.044303E+01	1.539821E+02	-6.573991E+01	1.717251E-15	3.789920E-14	9.657362E-16
7	1.0000	-1.036766E+01	-1.036756E+01	1.036766E+01	5.858938E-15	-5.911476E-15	1.911048E-16

ROW 5, X= .950, AXIAL LOAD= 2.944223E-11, BENDING MOMENT= 0.
DISPLACEMENT SOLUTION FOR PA=2.625000E+00, PB=0. WRITTEN ON TAPE18.
DISPLACEMENT SOLUTION FOR PA=2.812500E+00, PB=0. WRITTEN ON TAPE18.
DISPLACEMENT SOLUTION FOR PA=3.000000E+00, PB=0. WRITTEN ON TAPE18.
CP SECONDS= 26.677. NR OF IO REQUESTS (TAPE2)= 412. WORDS USED (TAPE2)= 44493. WORDS TRANSFERRED (TAPE2)= 1656666

Fig. 7-34 (Cont.)

Section 8
STAGS POST PROCESSOR

8.1 INTRODUCTION

The STAGS Post Processor (STAGPP) is designed to be used with the STAGS program. The function of the STAGS Post Processor is to present the calculated and stored data of STAGS in a two dimensional or contour plot output. The STAGS Post Processor takes the data stored on tape 19 by the STAGS program and creates a printer plot, a CAL-COMP pen plot, or both depending on the requirements of the user. The STAGS Post Processor is designed to plot selected output and/or contour plots of displacements (w, v, u, β_X, β_Y), and stress resultants (N_X, N_Y, N_{XY}, M_X, M_Y, M_{XY}). Also, the load displacement history and collapse mode plots are included in the post processor's capabilities.

This version of the STAGS Post Processor was written for the CDC 6600 computer utilizing a model 763, zip mode 10" drum CAL-COMP pen plotter. The user must supply the appropriate computer system controls when a CAL-COMP plot is requested.

There are two main branches of the STAGS Post Processor. The first branch allows for two dimensional plots of displacements, stress resultants and load displacement history and the second branch does only the contour plots of displacements, stress resultants, and the collapse mode. Figure 8-1 shows a typical data deck format. Table 8-1 is a summary of the input cards.

```
P1      0    2                          Linear Solution
P2      0    1    0    0
P1      1    2
P2      0    1    1    0
P3      1    1
P4    777                               Selected Displacements with Editing
P5    0.0
P6      1    1    1    0    0    0
P1      2    2
P2      0    1    1    0
P3      1    1
P4      1                               Selected Stress Resultants with
P5  777.0                               Editing
P6      1    1    1    0    0    0
P1      3    2                          Load Displacement History -
P2      0    1    0    0                Nonlinear
P1      4    2
P7     11   13    7   12    7   11   4  Collapse Mode Contour - Nonlinear
P8      1   13  131  122   82   79   1
P9      1    0    0    0    0    0
P10  CYL C/CUTOUT  COLLAPSE MODE
P1      5    2
P7     11   13    7   12    7   11   4  Special Output -
P8      1   13  131  122   82   79   1  Displacement Contour
P9      1    0    0    0    0    0
P10  CYL W/CUTOUT  W DISPLACEMENTS
P1      6    2
P7     11   13    7   12    1    4   7  Special Output -
P8      1    7   28   32  131  121   1  Stress Resultant Contour
P9      1    0    0    0    0    0
P10  CYL W/CUTOUT  NX STRESS RES.
P1      6    2
P7     11   13    5   12    1    4   7  Special Output -
P8     35   40   95   90   35           Stress Resultant Edited
P9      1    0    0    0    0    0
P10  CYL W/RECT CUTOUT (EDITED)
P1    999  999                          Normal Terminator for STAGS
                                        Post Processor
```

Fig. 8-1 STAGS Post Processor Sample Data Input (Sample Case 3, Page 7-22)

Table 8-1

STAGS POST PROCESSOR

SUMMARY OF INPUT CARDS

Card	Symbol	Format
P1	IPLT, IPLTSW	16I5
	Include Card P2 only if $0 \le$ IPLT ≤ 3 or IPLT = 7 or 8. If $4 \le$ IPLT ≤ 6 go to the P7 card.	
P2	IAXIAL, IOUT, IEDIT, IHST	16I5
	Include cards P3 through P6 only if IEDIT = 1 on the P2 card.	
P3	NSTEP, NXYEDT	16I5
P4	JSTEP(I), I=1, NSTEP	16I5
P5	XYEDIT(I), I=1, NXYEDT	8F10.5
P6	IVAR(I), I=1, NXYEDT	8F10.5
P6	IVAR(I), I=1,6	16I5
	Include cards P7 through P10 only if $4 \le$ IPLT ≤ 6 on the P1 card.	
P7	NROW, NCOL, NPB, NCTOUT, NRW1, NRW2, NCL1	16I5
P8	NB(I), I=1, NPB	16I5
P9	IVAR(I), I=1,6	16I5
P10	HEAD(I), I=1,3	3A10
	Include as many P10 cards as there are IVAR(I) = 1 on the P9 card.	

8.2 INPUT DESCRIPTION

P1 Plot Data Card

This card is used to select how and which stored data is to be plotted.

Variable	Format	Columns	Description
IPLT	I5	1-5	IPLT selects from a file created by STAGS the data which is to be plotted. IPLT=0 Plot linear solution. IPLT=1 Plot selected displacements. IPLT=2 Plot selected stress resultants. IPLT=3 Plot load displacement history (nonlinear analysis only) IPLT=4 Plot collapse mode (nonlinear only) IPLT=5 Displacements contour plot (special output see Page 6-33) IPLT=6 Stress resultants contour plot (special output). IPLT=7 Displacements plot (special output). IPLT=8 Stress resultants plot (special output). IPLT=999 Normal terminator for STAGS Post Processor.
IPLTSW	I5	6-10	IPLTSW=1 Do both printer and CAL-COMP pen plots. IPLTSW=2 Do only printer plots. IPLTSW=3 Do only CAL-COMP pen plots. IPLTSW=999 Normal terminator for STAGS Post Processor.

NOTE: If $4 \leq$ IPLT ≤ 6 go to the P7 card.

P2 Select Data Card

The select data card allows the user to exercise certain options concerning which load to select, print output, editing capability and load displacement history.

Variable	Format	Columns	Description	
IAXIAL	I5	1-5	IAXIAL=0	Plot load factor, PA.
			IAXIAL=1	Plot load factor, PB.
IOUT	I5	6-10	IOUT=0	Do not print plotted data.
			IOUT=1	Print plotted data.
IEDIT	I5	11-15	IEDIT=0	Plot everything requested on the P1 card. No more cards are needed.
			IEDIT=1	Plot only selected or special output requested on the P3 to P6 cards.
IHST	I5	16-20	IHST=0	Do not plot a load displacement history.
			IHST=1	If IPLT=3 do a CAL-COMP load displacement history pen plot.

P3 Edit Control Card

Used only if IEDIT=1 on the P2 card. The edit control card states how many load steps and/or rows or columns to select for plotting.

Variable	Format	Columns	Description
NSTEP	I5	1-5	NSTEP is the number of load steps requested. If no load steps are requested set NSTEP=1 and see card P4.
NXYEDT	I5	6-10	NXYEDT is the number of rows or columns requested. If no rows or columns are requested set NXYEDT=1 and see card P5.

P4 Select Individual Step Card

Used only if IEDIT=1 on the P2 card. This card specifies the individual load step requested for plotting.

Variable	Format	Columns	Description	
JSTEP(I) I=1, NSTEP	16I5	1-80	Actual load step.	
			JSTEP(I)=777	No editing by load step performed (use for linear and bifurcation analysis).
			JSTEP(I)=	Actual load step requested (use for nonlinear analysis only).

P5 Select Row or Column Card

Used only if IEDIT=1 on the P2 card. This card specifies the individual rows and columns requested for plotting.

Variable	Format	Columns	Description	
XYEDIT(I) I=1,NXYEDT	8F10.5	1-80	Actual rows or columns.	
			XYEDIT(1)=777.0	No editing by rows or columns performed.
			XYEDIT(I)=	Actual coordinate of rows or columns requested.

P6 Select Displacement or Stress Resultant Card

Used only if IEDIT=1 on the P2 card. This card reads data into an array which indicates which displacements or stress resultants are required. Displacements are requested if IPLT=1 or 7 on the P1 card. Stress resultants are requested if IPLT=2 or 8 on the P1 card.

Variable	Format	Columns	Description
IVAR(I) I=1,6	6I5	1-30	IVAR(I)=0 Do not plot IVAR(I)=1 Plot
			I=1 W displacement or NX stress resultant
			I=2 V displacement or NY stress resultant
			I=3 U displacement or NXY stress resultant
			I=4 BX displacement or MX stress resultant
			I=5 BY displacement or MY stress resultant
			I=6 MXY stress resultant

NOTE: Include cards P7 through P10 only if $4 \leq$ IPLT ≤ 6 on the P1 card.

P7 Contour Control Card

The contour control card allows the user to define the grid for contour plots. NROW and NCOLS (NR and NC in STAGS) define the grid and NPB is the number of points defining the plot boundaries stored in the array NB. The NROW and NCOL define the NB grid points where $1 \leq I \leq$ NROW and $1 \leq J \leq$ NCOL. Then NB = (I-1)*NCOL) + J (see Fig. 8-2). If a cutout is present NB will not include the grid points of the cutout (see Fig. 8-3). Once the NB array has been established the user can select a section of the total shell by choosing the proper NB values. Fig. 8-2 where (NB(I), I=1, NPB) = 7, 9, 14, 12, 7 will create a contour plot of the center portion of the flat plate only.

If IPLT=6 on the P1 card and NCOL > NROW on the P7 card, refer to Figs. 8-4 and 8-5 as examples of NCTOUT, NRW1, NRW2, and NCL1.

Variable	Format	Columns	Description
NROW	I5	1-5	Number of rows (stations along the X coordinate).
NCOL	I5	6-10	Number of columns (stations along the Y coordinate).
NPB	I5	11-15	Number of points stored in array NB which define the plot boundary.
NCTOUT	I5	16-20	Number of grid points of the cutout to be excluded (see Fig. 8-3).
NRW1	I5	21-25	Row number of one edge of cutout (same as in the STAGS program).
NRW2	I5	26-30	Row number of other edge of cutout (same as in the STAGS program).
NCL1	I5	31-35	Column number of edge of cutout (same as in the STAGS program)

P8 Plot Grid Card

This card reads the NB numbers which specify the contour plot boundary. Successive pairs of points listed in NB will be joined by a straight line. Note the first point in NB should be repeated as the last point in NB in order to close the contour plot.

Variable	Format	Column	Description
NB(I) I=1, NPB	10I5	1-50	The actual NB points which define the plot boundary (see Figs. 8-2 through 8-5).

P9 Select Displacement or Stress Resultant Card

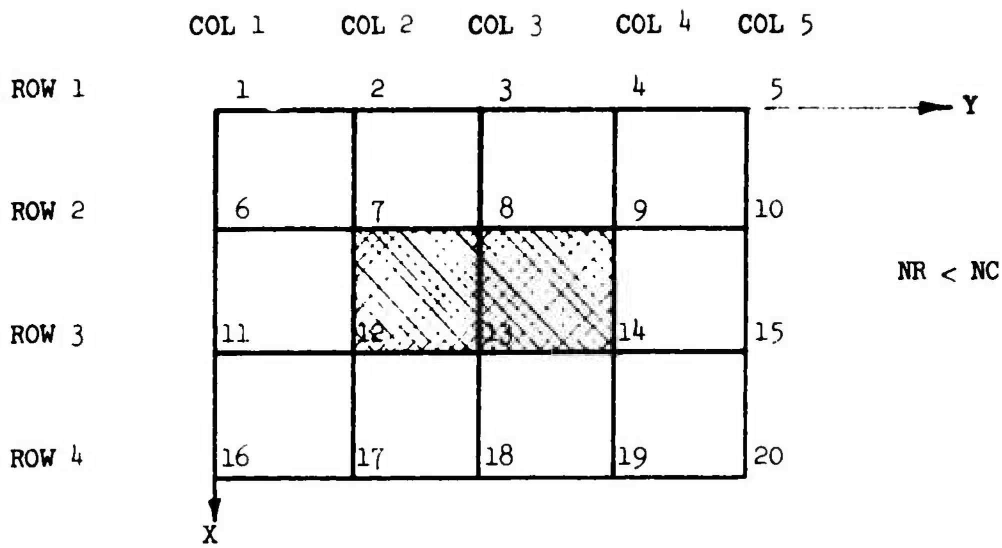
This card reads data into an array which indicates which displacements or stress resultants are required. If IPLT=5 on the P1 card, we have a contour plot of displacement. If IPLT=6 on the P1 card, we have a contour plot of stress resultant.

Variable	Format	Columns	Description
IVAR(I) I=1,6	6I5	1-30	IVAR(I)=0 Do not plot IVAR(I)=1 Plot
			I=1 W displacement or NX stress resultant.
			I=2 V displacement or NY stress resultant.
			I=3 U displacement or NXY stress resultant.
			I=4 MX stress resultant.
			I=5 MY stress resultant.
			I=6 MXY stress resultant.

P10 Plot Heading Card

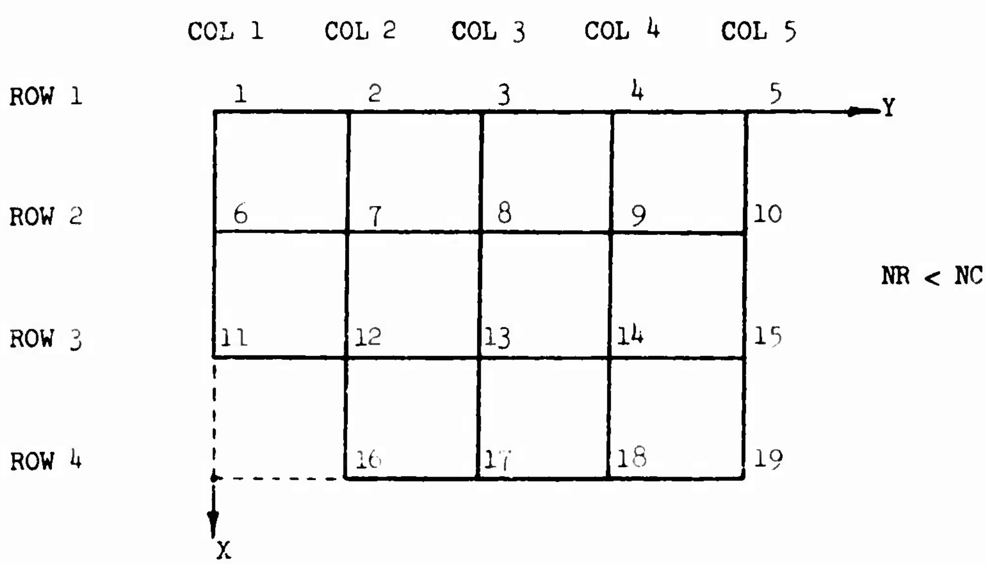
The user must read individual titles for each contour plot. There are as many P10 cards as there are IVAR(I) requested on the P9 card.

Variable	Format	Colums	Description
HEAD(I) I=1,3	3A10	1-50	User supplied heading for contour plot.



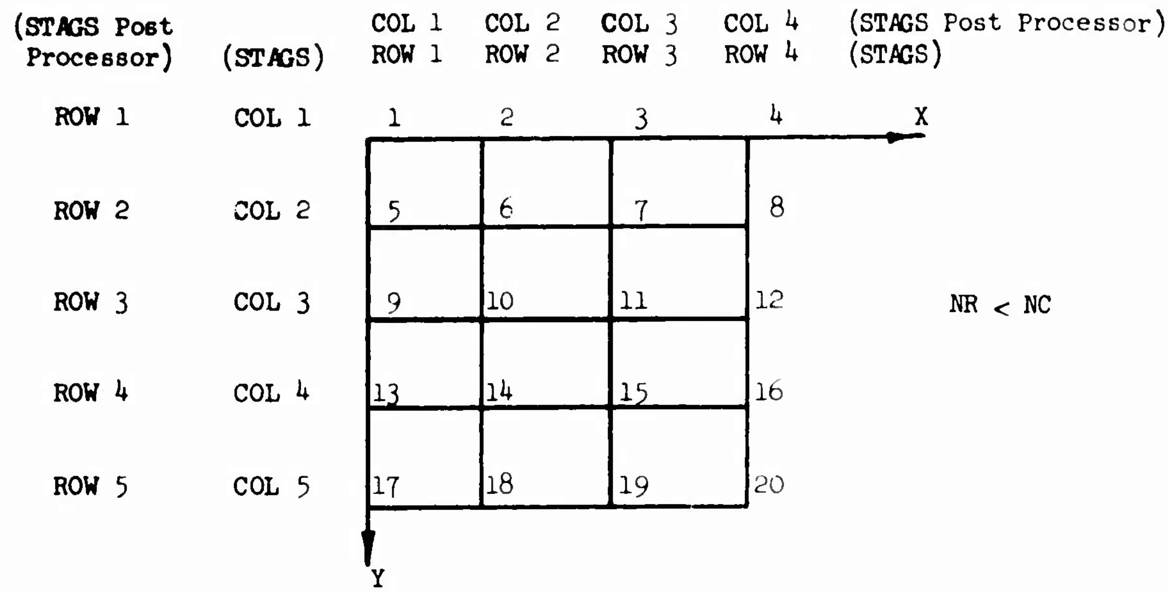
NROW=4, NCOL=5, NPB=5, NCTOUT=0, NRW1=0, NRW2=0, NCL1=0, (NB(I),I=1,NPB) =
{NB:1,5,20,16,1}

Fig. 8-2 Flat Plate Contour Control Card Setup



NROW=4, NCOL=5, NPB=7, NCTOUT=1, NRW1=3, NRW2=4, NCL1=2, (NB(I),I=1,NPB) =
{NB:1,5,19,16,12,11,1}

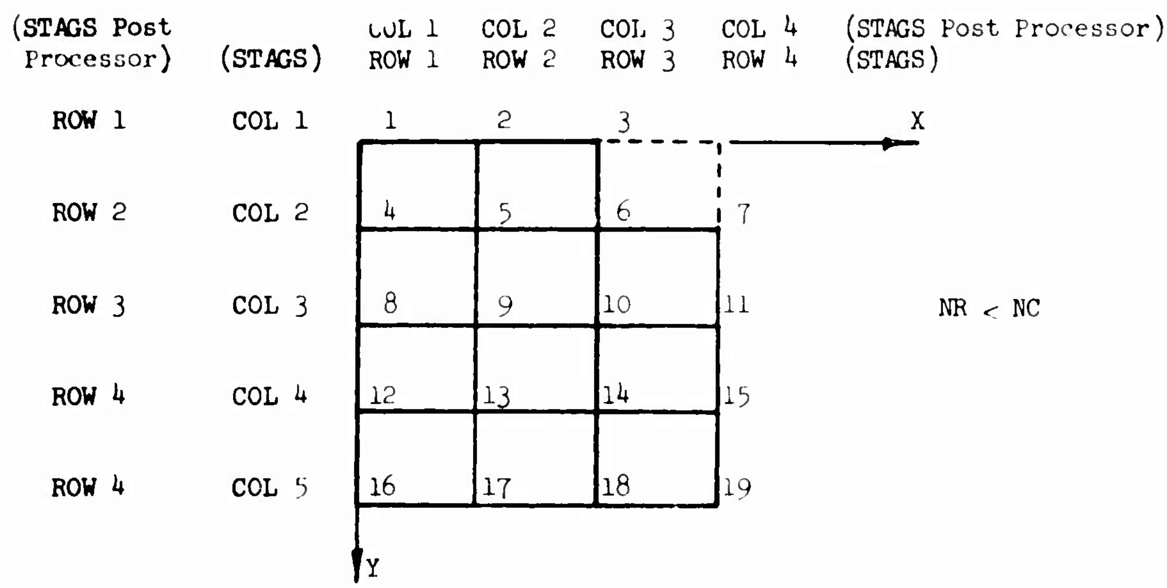
Fig. 8-3 Flat Plate with Cutout Contour Control Card Setup



NROW=4, NCOL=5, NPB=5, NCTOUT=0, NRW1=0, NRW2=0, NCL1=0, (NB(I),I=1,NPB)

{NB:1,4,20,17,1}

Fig. 8-4 Flat Plate Contour Control Card Setup When NCOL Greater Than NROW (Stress Resultants Only)



NOTE: NRW1, NRW2, NCL1 are in the STAGS Post Processor Notation

NROW=4, NCOL=5, NPB=7, NCTOUT=1, NRW1=1, NRW2=2, NCL1=3, (NB(I),I=1,(NPB) =

{MB:1,3,6,7,19,16,1}

Fig. 8-5 Flat Plate with Cutout Contour Control Card Setup When NCOL Greater Than NROW (Stress Resultants Only)

Section 9
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